Day 11 **S Tier 10-a-day: Model Solutions**

1 Arc length and sector area

A sector of a circle has angle 120° and area 12π cm^2.
(a) Work out the radius of the circle.
(b) Work out the length of the arc of the sector. Give your answer to (b) in terms of π.

(a) Area = $\frac{120}{360} \times \pi \times r^2$, so $\frac{120}{360} \times r^2 = 12$

$r^2 = 12 \times \frac{360}{120} = 36$

$r = 6$ cm

(b) Arc length = $\frac{120}{360} \times 2 \times \pi \times 6 = 4\pi$ cm

Answer: (a) 6 cm (b) 4π cm

2 Factorising quadratics

Factorise $6x^2 - 11x - 10$

$6 \times (-10) = -60$. Two numbers that multiply to -60 and add to -11 are -15 and 4
Split the middle term: $6x^2 - 15x + 4x - 10$
Factorise in pairs: $3x(2x - 5) + 2(2x - 5)$
$= (3x + 2)(2x - 5)$
Answer: $(3x + 2)(2x - 5)$

3 Averages with algebra

Here are five numbers: $x - 2$, $3x + 3$, $x - 3$, $3x + 4$, $2x - 2$.
The mean of the five numbers is 6.
(a) Work out the value of x.
(b) Work out the range of the five numbers.

(a) Total = $(x - 2) + (3x + 3) + (x - 3) + (3x + 4) + (2x - 2) = 6 \times 5 = 30$
$10x + 0 = 30$, so $10x = 30$ and $x = 3$
The numbers are 1, 12, 0, 13, 4
(b) Range = $13 - 0 = 13$
Answer: (a) x = 3 (b) 13

4 Percentage change

The number of visitors is decreased by 50%.
By what percentage must the new value be increased to get back to the original value?

Let the original be 100. After a 50% decrease it is 50.
To return to 100, increase by 50

Percentage increase = $\frac{50}{50} \times 100 = 100\%$

Answer: 100%

5 Where a line meets the axes

The line L has equation $15x - 8y = 120$.
L crosses the x-axis at the point A and the y-axis at the point B.
(a) Find the coordinates of A and B.
(b) Work out the area of triangle OAB, where O is the origin.

(a) At A, $y = 0$: $15x = 120$, so $x = 8$. A is (8, 0)
At B, $x = 0$: $-8y = 120$, so $y = -15$. B is (0, -15)
(b) OA = 8 and OB = 15, and angle AOB = 90°

Area = $\frac{1}{2} \times 8 \times 15 = 60$

Answer: (a) A(8, 0), B(0, -15) (b) 60

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6 Laws of indices

Given that $\frac{16^5 \times 32^4}{4^2} = 2^n$,

find the value of n.

$16 = 2^4$, $32 = 2^5$, $4 = 2^2$

Numerator: $2^{20} \times 2^{20} = 2^{40}$

Denominator: 2^4

Subtract the powers: $40 - 4 = 36$

Answer: n = 36

7 Solving trigonometric equations

Solve $\tan x = 1$ for $0° \leq x \leq 360°$.

Do not use a calculator.

The related acute angle satisfies $\tan \theta = 1$, so $\theta = 45°$

tangent is positive in the 1st and 3rd quadrants, so $x = 45°$ and $x = 180° + 45°$

$x = 45°$ and $x = 225°$

Answer: x = 45° or x = 225°

8 Quadratic formula and surds

Solve $4x^2 + x - 2 = 0$

Give your answers in surd form.

Use the quadratic formula with $a = 4$, $b = 1$, $c = -2$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 4 \times (-2)}}{2 \times 4}$$

$$x = \frac{-1 \pm \sqrt{33}}{8}$$

Answer: $\mathbf{x = \frac{-1 \pm \sqrt{33}}{8}}$

9 Squares, cubes and prime factors

Find the smallest positive integer k such that $560 \div k$ is a square number.

$560 = 2^4 \times 5 \times 7$

For a square number every power must be even

Divide by 5×7 to get 2^4

$k = 35$

Answer: k = 35

10 Probability and quadratic equations

A bag contains some counters. 5 of them are green and the rest are white.

Two counters are taken at random without replacement.

The probability that both are green is $\frac{2}{11}$.

Work out how many counters are white.

Let there be n counters altogether

$$P(\text{both green}) = \frac{5}{n} \times \frac{4}{n-1} = \frac{20}{n(n-1)}$$

$\frac{20}{n(n-1)} = \frac{2}{11}$, so $n(n-1) = 110$, giving $n^2 - n - 110 = 0$

$(n - 11)(n + 10) = 0$, so $n = 11$ (n cannot be negative)

Number of counters that are white $= 11 - 5 = 6$

Answer: 6