Day 109 **S Tier 10-a-day: Model Solutions**

1 Solving trigonometric equations

Solve $7 \sin x - 6 = 0$ for $0° \leq x \leq 360°$. (Calculator)
Give your answers correct to 1 decimal place.

$\sin x = \frac{6}{7}$

Related acute angle: $\sin^{-1}(\frac{6}{7}) = 59.0°$

sine is positive in the 1st and 2nd quadrants, so $x = 59.0°$ and $x = 180° - 59.0°$
$x = 59.0°$ and $x = 121.0°$
Answer: $x = 59.0°$ or $x = 121.0°$

2 Solving quadratics by factorising

Solve $4x^2 + 14 = -15x$

Rearrange so that one side is 0: $4x^2 + 15x + 14 = 0$
Factorise: $(4x + 7)(x + 2) = 0$
$4x + 7 = 0$ or $x + 2 = 0$
$x = -\frac{7}{4}$ or $x = -2$

Answer: $x = -2$ or $x = -\frac{7}{4}$

3 HCF and LCM

The HCF of two numbers is 5 and their LCM is 70.
The smaller number is 10.
Work out the larger number.

HCF × LCM = product of the two numbers
$5 \times 70 = 350$
Larger number $= 350 \div 10 = 35$
Answer: 35

4 Inverse proportion

P is inversely proportional to the cube of t.
When $t = 4$, $P = 2$.
Find the positive value of t when $P = 16$.

$P = \frac{k}{t^3}$

$2 = \frac{k}{64}$, so $k = 2 \times 64 = 128$

$16 = \frac{128}{t^3}$, so $t^3 = \frac{128}{16} = 8$

$t = \sqrt[3]{8} = 2$
Answer: $t = 2$

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5 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 4. N is on AB with AN : NB = 1 : 3.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find MN in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{1}{4}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{1}{4}(\mathbf{b} - \mathbf{a}) = \frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$

(b) OM = $\frac{1}{5}\mathbf{a}$

MN = ON − OM = $\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b} - \frac{1}{5}\mathbf{a} = \frac{11}{20}\mathbf{a} + \frac{1}{4}\mathbf{b}$

Answer: (a) $\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$ (b) $\frac{11}{20}\mathbf{a} + \frac{1}{4}\mathbf{b}$

6 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve. It crosses the axes at the points shown.
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The roots are $x = -1$ and $x = 5$, so $y = a(x + 1)(x - 5)$
When $x = 0$, $y = 10$: $a \times 1 \times (-5) = 10$, so $a = -2$
$y = -2(x + 1)(x - 5)$
$y = -2x^2 + 8x + 10$
Answer: $y = -2x^2 + 8x + 10$

7 Index equations

Solve $2^{2x-2} = 8^4$

Write both sides as powers of 2: $8 = 2^3$
Equate the powers: $2x - 2 = 12$
$2x = 14$, so $x = 7$
Answer: $x = 7$

8 Percentages with an unknown rate

A population is increased by x% in one year and then increased by a further x% the next year.
Over the two years the population increases by 125%.
Work out the value of x.

Yearly multiplier = $1 + \frac{x}{100}$

Two years: $(1 + \frac{x}{100})^2 = 2.25$

Take the square root: $1 + \frac{x}{100} = 1.5$

$\frac{x}{100} = 0.5$, so $x = 50$
Answer: $x = 50$

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9 Simultaneous equations

Solve the simultaneous equations

$-3x + 4y = 31$

$-3x + 6y = 45$

To match the y terms: (1) × 3 gives $-9x + 12y = 93$, and (2) × 2 gives $-6x + 12y = 90$

Subtract the equations to eliminate y: $-3x = 3$

$x = -1$

Substitute $x = -1$ into (1): $4y + 3 = 31$

$4y = 28$, so $y = 7$

Answer: $x = -1$, $y = 7$

10 Winning a match

Isla and Carlos play a match. The first to win 2 table tennis games wins the match.

The probability that Isla wins any one of the table tennis games is $\frac{3}{10}$.

Work out the probability that Isla wins the match.

Isla wins by WW (2 games) or WLW or LWW (3 games), where W means Isla wins that game

$P(WW) = \frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$

$P(WLW) = P(LWW) = \frac{3}{10} \times \frac{7}{10} \times \frac{3}{10} = \frac{63}{1000}$

$\text{Total} = \frac{9}{100} + 2 \times \frac{63}{1000} = \frac{27}{125}$

Answer: $\frac{27}{125}$