Day 108 **S Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $(p - q)^2 - (p + q)^2$
when $p = -2$, $q = 3$ and $r = 3$.

Substitute the values: $(-2 - 3)^2 - (-2 + 3)^2$
$= 25 - 1$
$= 24$
Answer: 24

2 Venn diagrams with algebra

The Venn diagram shows how many of 27 students play the piano (P) or the guitar (G).
(a) Work out the value of x.
(b) A student is chosen at random from those who play the guitar. Work out the probability that this student also plays the piano.

(a) Total: $(x + 3) + x + (2x) + 8 = 27$
$4x + 11 = 27$, so $x = 4$
(b) Students in the second circle: $x + 2x = 4 + 8 = 12$, of whom 4 are in both
Probability = $\frac{1}{3}$

Answer: (a) x = 4 (b) $\frac{1}{3}$

3 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 21°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 21°
Angle COB is an exterior angle of triangle OAC: 21 + 21 = 42°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 42°

4 Mean of a changing set

The mean of 11 numbers is 99.
One number is removed and the mean of the remaining 10 numbers is 107.2.
Work out the number that was removed.

Total of the 11 numbers = 11 × 99 = 1089
Total of the 10 remaining = 10 × 107.2 = 1072
Number removed = 1089 − 1072 = 17
Answer: 17

5 Loci and regions

ABCD is a rectangle with AB = 6 cm and AD = 6 cm.
A point P inside the rectangle is such that AP ≤ 4 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 4 cm: P is inside the quarter circle of radius 4 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves
Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 4^2 = 2\pi$ cm^2

Answer: 2π cm^2

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6 Simple and compound interest

Sofia invests £10,000 for 3 years at 2% per year simple interest.
Jess invests £10,000 for 3 years at 2% per year compound interest.
Work out how much more interest Jess earns than Sofia.
Give your answer to the nearest penny. (Calculator)

Simple interest: £10,000 × 2% × 3 = £600

Compound: £10,000 × 1.02^3 = £10,612.08

Compound interest = £10,612.08 − £10,000 = £612.08

Difference = £612.08 − £600 = £12.08

Answer: £12.08

7 Estimating the median from grouped data

The table gives information about 94 times.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 21, 40, 60, 86, 94

The median is the value at position 94 ÷ 2 = 47

It is in the class 30 to 40, with 40 values before it, so we need 7 more of the 20 values in this class

Median ≈ $30 + \frac{7}{20} \times 10 = 33.5$

Answer: 33.5

8 Fractional indices

Given that x > 0, solve $x^{\frac{1}{3}} = 3$

Raise both sides to the power 3

$x = (3)^3$

$x = 27$

Answer: x = 27

9 Algebraic proof (harder)

Show algebraically that the sum of the cubes of any three consecutive integers is a multiple of 9.

$(m - 1)^3 + m^3 + (m + 1)^3 = 3m^3 + 6m = 3m(m^2 + 2)$

If m is a multiple of 3, then $3m(m^2 + 2)$ is a multiple of 9

If $m = 3m + 1$ or $m = 3m - 1$, then $m^2 + 2 = 9m^2 \pm 6m + 3 = 3(3m^2 \pm 2m + 1)$, a multiple of 3

In every case $3m(m^2 + 2)$ contains two factors of 3, so it is a multiple of 9

Answer: Proved

10 Surds with unknowns

Given that $(a + \sqrt{14})^2 = 39 + 10\sqrt{14}$,
where a is a positive integer, find the value of a.

$(a + \sqrt{14})^2 = a^2 + 2a\sqrt{14} + 14$

Compare the $\sqrt{14}$ terms: 2a = 10, so a = 5

Check the integer terms: $a^2 + 14 = 25 + 14 = 39$

Answer: a = 5