Day 107 **S Tier 10-a-day: Model Solutions**

1 Arc length and sector area

A sector of a circle has angle 45° and area $\frac{121}{8}\pi$ cm^2.

(a) Work out the radius of the circle.

(b) Work out the length of the arc of the sector. Give your answer to (b) in terms of π.

(a) Area = $\frac{45}{360} \times \pi \times r^2$, so $\frac{45}{360} \times r^2 = \frac{121}{8}$

$r^2 = \frac{121}{8} \times \frac{360}{45} = 121$

$r = 11$ cm

(b) Arc length = $\frac{45}{360} \times 2 \times \pi \times 11 = \frac{11}{4}\pi$ cm

Answer: (a) 11 cm (b) $\frac{11}{4}\pi$ cm

2 Inequalities

(a) Solve $11 - 3x < 14$

(b) n is an integer. Write down all the values of n that satisfy

$-1 < 3n - 4 \leq 17$

(a) Subtract 11: $-3x < 3$

Divide by -3 and reverse the inequality sign: $x > -1$

(b) Add 4: $3 < 3n \leq 21$

Divide by 3: $1 < n \leq 7$

Integers: 2, 3, 4, 5, 6, 7

Answer: (a) $x > -1$ (b) 2, 3, 4, 5, 6, 7

3 Weighted percentages

The ratio of Year 10 students to Year 11 students in a school is 1 : 3.

65% of the Year 10 students and 45% of the Year 11 students have a part-time job.

What percentage of all of these students have a part-time job?

Imagine 80 students in total: 20 Year 10 students and 60 Year 11 students

65% of 20 = 13 and 45% of 60 = 27

Together: 40 out of 80

40 ÷ 80 × 100 = 50%

Answer: 50%

4 Expanding three brackets

Expand and simplify $(x + 6)(x + 4)(x - 2)$

Expand the first two brackets: $(x + 6)(x + 4) = x^2 + 10x + 24$

Multiply by $(x - 2)$: $x(x^2 + 10x + 24) - 2(x^2 + 10x + 24)$

$= x^3 + 8x^2 + 4x - 48$

Answer: $x^3 + 8x^2 + 4x - 48$

5 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 2 cm, DB = 4 cm and the area of triangle ADE is 9 cm^2.

Work out the area of the trapezium DBCE.

The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 2 + 4 = 6 cm

Length scale factor = 6 ÷ 2 = 3

Area scale factor = $(3)^2 = 9$

Area of triangle ABC = 9 × 9 = 81 cm^2

Trapezium = 81 − 9 = 72 cm^2

Answer: 72 cm^2

Day 107 **S Tier 10-a-day: Model Solutions**

6 Percentage change in area and volume

The edge length of a cube is increased by 60%.
Work out the percentage change in the volume of the cube.

Side length multiplier = 1.6

Volume multiplier = $1.6^3 = 4.096$

$4.096 - 1 = 3.096$, so a 309.6% increase

Answer: 309.6% increase

7 Tangent to a circle

A circle has equation $(x + 1)^2 + (y - 3)^2 = 169$.
The point $P(-6, 15)$ lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form $ax + by = c$, where a, b and c are integers.

The centre is $(-1, 3)$. The gradient of the radius to P is $\frac{15 - 3}{-6 - (-1)} = -\frac{12}{5}$

The tangent is perpendicular to the radius, so its gradient is $\frac{5}{12}$. Using P: $y - 15 = \frac{5}{12}(x + 6)$

Multiply by 12: $12(y - 15) = 5(x + 6)$

Expand and rearrange: $-5x + 12y = 210$

Multiply by -1: $5x - 12y = -210$

Answer: $5x - 12y = -210$

8 Multiplying and dividing surds

Simplify fully $4\sqrt{8} \times 3\sqrt{3}$

$4\sqrt{8} \times 3\sqrt{3} = 12\sqrt{24}$

$\sqrt{24} = 2\sqrt{6}$, so $12 \times 2\sqrt{6} = 24\sqrt{6}$

Answer: $24\sqrt{6}$

9 Exact trigonometric ratios from a given ratio

x is an acute angle and $\tan x = \frac{5}{6}$.

(a) Find the exact value of sin x.
(b) Find the exact value of cos x. Give each answer as a fraction or a surd with a rational denominator.

Draw a right-angled triangle with opposite side 5 and adjacent side 6

Hypotenuse $= \sqrt{5^2 + 6^2} = \sqrt{61}$

(a) $\sin x = \frac{5}{\sqrt{61}} = \frac{5\sqrt{61}}{61}$

(b) $\cos x = \frac{6}{\sqrt{61}} = \frac{6\sqrt{61}}{61}$

Answer: (a) $\frac{5\sqrt{61}}{61}$ (b) $\frac{6\sqrt{61}}{61}$

Day 107 **S Tier 10-a-day: Model Solutions**

10 Probability and quadratic equations

A bag contains some beads. 3 of them are blue and the rest are black.
Two beads are taken at random without replacement.

The probability that both are blue is $\frac{3}{10}$.

Work out how many beads are black.

Let there be n beads altogether

P(both blue) = $\frac{3}{n} \times \frac{2}{n-1} = \frac{6}{n(n-1)}$

$\frac{6}{n(n-1)} = \frac{3}{10}$, so $n(n-1) = 20$, giving $n^2 - n - 20 = 0$

$(n-5)(n+4) = 0$, so $n = 5$ (n cannot be negative)

Number of beads that are black = $5 - 3 = 2$

Answer: 2