Day 105 **S Tier 10-a-day: Model Solutions**

1 Compound interest

Evan invests £11,000 at 6% per year compound interest.
After how many whole years will the investment first be worth more than £13,600?
(Calculator)

Multiplier = 1.06

After 3 years: $11000 \times 1.06^3 = 13101.18$

After 4 years: $11000 \times 1.06^4 = 13887.25$

13887.25 is the first value above £13,600

Answer: 4 years

2 Rationalising the denominator

Rationalise the denominator of $\frac{5}{3\sqrt{3}}$

Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{3}$:

$\frac{5}{3\sqrt{3}} = \frac{5\sqrt{3}}{9}$

Answer: $\frac{5\sqrt{3}}{9}$

3 Inverse proportion in context

8 painters can paint a hall in 10 days.
After 3 days, 6 of the painters leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = $8 \times 10 = 80$ painter-days

Work done = $8 \times 3 = 24$, so work left = 56

Workers left = $8 - 6 = 2$

Time = $56 \div 2 = 28$ days

Answer: 28 days

4 Percentage change

The population of a village decreased by 5% in 2023.
In 2024 the population decreased by a further 10%.
Work out the overall percentage decrease over the two years.

Multipliers: 0.95 and 0.9

Overall multiplier = $0.95 \times 0.9 = 0.855$

$0.855 - 1 = -0.145$, so an overall decrease of 14.5%

Answer: 14.5% decrease

5 Quadratic formula and surds

Solve $2x^2 - 4x - 1 = 0$ (Calculator)
Give your answers correct to 2 decimal places.

Use the quadratic formula with $a = 2$, $b = -4$, $c = -1$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 2 \times (-1)}}{2 \times 2} = \frac{4 \pm \sqrt{24}}{4}$

$x = -0.2247...$ or $x = 2.2247...$

$x = -0.22$ or $x = 2.22$ (2 d.p.)

Answer: $x = -0.22$ or $x = 2.22$

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6 3D problems: pyramids

VABCD is a pyramid on a square base. V is directly above the centre M.
(a) Work out the height VM of the pyramid.
(b) Work out the volume of the pyramid.
The diagram is not drawn accurately.

(a) $AC^2 = 16^2 + 16^2 = 512$, so $AM^2 = (AC \div 2)^2 = 128$
$VM^2 = VA^2 - AM^2 = 12^2 - 128 = 16$
VM = 4 cm
(b) Volume = $\frac{1}{3} \times 16^2 \times 4 = \frac{1024}{3}$ cm^3

Answer: (a) 4 cm (b) $\frac{1024}{3}$ cm^3

7 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and a > 0.
The curve passes through the points (1, 12) and (4, 96).
Find the value of k and the value of a.

Substitute the points: $k\,a^1 = 12$ and $k\,a^4 = 96$
Divide: $a^3 = \frac{96}{12} = 8$
a = 2
From the first point: $k \times 2 = 12$, so k = 6
Answer: k = 6, a = 2

8 Cone made from a sector

A sector of a circle has radius 12 cm and angle 150°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{150}{360} \times 2\pi \times 12 = 10\pi$ cm
This is the circumference of the base: $2\pi r = 10\pi$, so r = 5 cm
(b) Slant height = 12 cm, so height$^2 = 12^2 - 5^2 = 119$, height = $\sqrt{119}$ cm
Volume = $\frac{1}{3} \times \pi \times 5^2 \times \sqrt{119} = \frac{25\sqrt{119}}{3}\pi$ cm^3

Answer: (a) 5 cm (b) $\frac{25\sqrt{119}}{3}\pi$ cm^3

9 Successive percentage change

The value of a car is increased by 5%.
The new amount is then decreased by 50%.
Work out the overall percentage change.

Multipliers: 1.05 and 0.5
1.05 × 0.5 = 0.525
0.525 is a decrease of 47.5%
Answer: 47.5% decrease

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10 Exact trigonometric ratios from a given ratio

x is an acute angle and $\tan x = \frac{1}{6}$.

(a) Find the exact value of sin x.

(b) Find the exact value of cos x. Give each answer as a fraction or a surd with a rational denominator.

Draw a right-angled triangle with opposite side 1 and adjacent side 6

Hypotenuse = $\sqrt{1^2 + 6^2} = \sqrt{37}$

(a) $\sin x = \frac{1}{\sqrt{37}} = \frac{\sqrt{37}}{37}$

(b) $\cos x = \frac{6}{\sqrt{37}} = \frac{6\sqrt{37}}{37}$

Answer: (a) $\frac{\sqrt{37}}{37}$ **(b)** $\frac{6\sqrt{37}}{37}$