Day 104 **S Tier 10-a-day: Model Solutions**

1 Successive percentage change

The value of a car is decreased by 50%.
The new amount is then decreased by 30%.
Work out the overall percentage change.

Multipliers: 0.5 and 0.7
0.5 × 0.7 = 0.35
0.35 is a decrease of 65%
Answer: 65% decrease

2 Laws of indices with algebra

Simplify fully $\frac{3x^6y^3 \times 3x^5y}{3x^2y}$

Give your answer in the form kx^ay^b

Numbers: 3 × 3 ÷ 3 = 3
Powers of x: 6 + 5 − 2 = 9
Powers of y: 3 + 1 − 1 = 3
$= 3x^9y^3$
Answer: $3x^9y^3$

3 Combining means

The mean of the heights in cm for the Year 10 students is 78.
The mean of the heights in cm for the Year 11 students is 63.
The mean for all of the Year 10 students and Year 11 students together is 71.75.
Work out the ratio of the number of Year 10 students to the number of Year 11 students.

Let there be p Year 10 students and q Year 11 students
Total = 78p + 63q, and the overall mean gives 78p + 63q = 71.75(p + q)
6.25p = 8.75q
p : q = 8.75 : 6.25 = 7 : 5
Answer: 7 : 5

4 Fractions of fractions

Dina spent $\frac{1}{3}$ of her money on a coat.

She then spent $\frac{4}{7}$ of the money that was left on a bag.

A bag cost £1,352.
How much money did Dina have to start with?

The second amount is $\frac{4}{7}$ of $\frac{2}{3}$, which is $\frac{8}{21}$ of the total

$\frac{8}{21}$ of the total is £1,352

Total = £1,352 ÷ $\frac{8}{21}$ = £3,549

Answer: £3,549

5 Recasting solids

A solid cylinder of radius 10 cm and height 9 cm is melted down.
The metal is used to make identical solid spheres of radius 3 cm.
Work out the number of spheres that can be made.

The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Volume of cylinder = $\pi \times 10^2 \times 9 = 900\pi$

Volume of one sphere = $\frac{4}{3} \times \pi \times 3^3 = 36\pi$

Number of spheres = 900 ÷ 36 = 25
Answer: 25

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6 Tangent to a circle

A circle has equation $x^2 + y^2 = 65$.
The point P(−8, −1) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is (0, 0). The gradient of the radius to P is $\frac{-1-0}{-8-0} = \frac{1}{8}$

The tangent is perpendicular to the radius, so its gradient is −8. Using P: y + 1 = −8(x + 8)
Expand and rearrange: 8x + y = −65
Answer: 8x + y = −65

7 Angles of elevation

A point A is on level ground in line with the base of a tower. (Calculator)
The angle of elevation of the top of the tower from A is 48°.
B is a point on the same line, 100 m further from the tower than A.
The angle of elevation of the top of the tower from B is 18°.
Work out the height of the tower. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 48° = \frac{h}{x}$, so x = h ÷ tan 48°

From B: $\tan 18° = \frac{h}{x + 100}$, so x + 100 = h ÷ tan 18°

Subtract: 100 = h ÷ tan 18° − h ÷ tan 48° = h(3.0777 − 0.90040)
h = 100 ÷ 2.1773 = 45.9 m (1 d.p.)
Answer: 45.9 m

8 Finding a percentage rate

A computer was bought for £5,200.
Each year its value decreases by the same percentage.
After 2 years it is worth £4,212.
Work out the percentage decrease each year.

Multiplier for 2 years = £4,212 ÷ £5,200 = 0.81
Yearly multiplier = the square root of 0.81 = 0.9
10% decrease each year
Answer: 10%

9 Finding the rate of interest

Nadia invested £15,500 in a savings account for 2 years.
Compound interest was paid at a fixed rate per year.
At the end of the 2 years the account held £17,415.80.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $15500 \times m^2 = 17415.80$
$m^2 = 17415.80 \div 15500 = 1.1236$
$m = \sqrt{1.1236} = 1.06$
Rate = (1.06 − 1) × 100 = 6%
Answer: 6% per year

10 Surds

Given that $\sqrt{252} - \sqrt{28} = \sqrt{n}$,
find the value of n.

$\sqrt{252} = 6\sqrt{7}$ and $\sqrt{28} = 2\sqrt{7}$
$6\sqrt{7} - 2\sqrt{7} = 4\sqrt{7}$
$4\sqrt{7} = \sqrt{112}$, so n = 112
Answer: n = 112