Day 103 **S Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $x^2 - 3yz$
when $x = -4$, $y = 2$ and $z = -4$.

Substitute the values: $(-4)^2 - 3 \times 2 \times (-4)$
$= 16 - (-24)$
$= 40$
Answer: 40

2 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 26 is taken. Work out how many come from Group A.
(b) A different stratified sample has 15 from Group B. Work out the size of this sample.

Total = 180 + 100 + 60 + 180 = 520

(a) Group A: $\frac{180}{520} \times 26 = 9$

(b) Fraction sampled = $15 \div 100 = \frac{3}{20}$, so sample size = $\frac{3}{20} \times 520 = 78$

Answer: (a) 9 (b) 78

3 Enlargement

Triangle P has vertices A(2, −3), B(1, −1) and C(4, −6).
Triangle Q has vertices A'(4, −3), B'(2, 1) and C'(8, −9).
Q is an enlargement of P.
Describe fully the single transformation that maps P onto Q.

x-step from A to B = −1, x-step from A' to B' = −2
Scale factor = $-2 \div -1 = 2$
The centre C satisfies: $C + 2(A - C) = A'$, so $(1 - 2)C = A' - 2A$
x: $(-1) \times x = 4 - 2 \times 2 = 0$, so $x = 0$
y: $(-1) \times y = (-3) - 2 \times (-3) = 3$, so $y = -3$
Answer: Enlargement, scale factor 2, centre (0, −3)

4 Equations with indices

Solve $x^{\frac{4}{3}} = 256$
$x > 0$

Write 256 as a power of 4: $256 = 4^4$
$x^{\frac{4}{3}} = 4^4$, so $x = (4^4)^{\frac{3}{4}}$
$x = 4^3 = 64$
Answer: x = 64

5 Surds

Given that $\sqrt{75} - \sqrt{12} = \sqrt{n}$,
find the value of n.

$\sqrt{75} = 5\sqrt{3}$ and $\sqrt{12} = 2\sqrt{3}$
$5\sqrt{3} - 2\sqrt{3} = 3\sqrt{3}$
$3\sqrt{3} = \sqrt{27}$, so n = 27
Answer: n = 27

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6 Perpendicular bisector

A is the point (4, −3) and B is the point (12, −9).
Find an equation of the perpendicular bisector of AB. Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = (8, −6)

Gradient of AB = $-\frac{3}{4}$

Perpendicular gradient = $\frac{4}{3}$

$y - (-6) = \frac{4}{3}(x - 8)$

Rearrange to integer coefficients: $4x - 3y = 50$

Answer: $4x - 3y = 50$

7 Direct proportion formulae

L is directly proportional to the square root of t.
When t = 9, L = 21.
(a) Find a formula for L in terms of t.
(b) Work out the value of L when t = 25.

(a) $L = k\sqrt{t}$
$21 = k \times 3$, so $k = 21 \div 3 = 7$
$L = 7\sqrt{t}$
(b) $L = 7 \times 5 = 35$

Answer: (a) $L = 7\sqrt{t}$ (b) 35

8 Algebraic proof

Prove that the sum of the squares of any three consecutive integers is always 2 more than a multiple of 3.

Let the integers be m − 1, m and m + 1.
$(m - 1)^2 + m^2 + (m + 1)^2 = m^2 - 2m + 1 + m^2 + m^2 + 2m + 1$
$= 3m^2 + 2$
$3m^2$ is a multiple of 3, so the sum is 2 more than a multiple of 3

Answer: Proved

9 Tangent to a circle

A circle has equation $x^2 + y^2 = 25$.
The point P(−4, 3) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $-\frac{3}{4}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div (-\frac{3}{4}) = \frac{4}{3}$

$y - 3 = \frac{4}{3}(x - (-4))$

Multiply out and rearrange: $4x - 3y = -25$
(b) On the x-axis y = 0, so $4x = -25$
$x = -\frac{25}{4}$, so T is $(-\frac{25}{4}, 0)$

Answer: (a) $4x - 3y = -25$ (b) T = $(-\frac{25}{4}, 0)$

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10 Counting factors

$N = 2^3 \times 3 \times 5^3$

How many of the factors of N are even numbers?

An even factor must contain at least one 2
Power of 2: 1 to 3, that is 3 choices
Power of 3: 0 to 1, that is 2 choices
Power of 5: 0 to 3, that is 4 choices
Total = $3 \times 2 \times 4 = 24$
Answer: 24