Day 102 **S Tier 10-a-day: Model Solutions**

1 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 54°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle APB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 54 = 108°
(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angle APB = 360 − 90 − 90 − 108 = 72°
Answer: (a) 108° (b) 72°

2 Mean of a changing set

The mean of 7 numbers is 20.
A new number is added and the mean of all 8 numbers is 25.
Work out the new number.

Total of the 7 numbers = 7 × 20 = 140
New total = 8 × 25 = 200
New number = 200 − 140 = 60
Answer: 60

3 Inequalities

(a) Solve $1 - 6x < -17$
(b) n is an integer. Write down all the values of n that satisfy
$3 < 3n + 3 \leq 24$

(a) Subtract 1: $-6x < -18$
Divide by -6 and reverse the inequality sign: $x > 3$
(b) Subtract 3: $0 < 3n \leq 21$
Divide by 3: $0 < n \leq 7$
Integers: 1, 2, 3, 4, 5, 6, 7
Answer: (a) $x > 3$ (b) 1, 2, 3, 4, 5, 6, 7

4 Rationalising the denominator

Write $\frac{2+\sqrt{6}}{3-\sqrt{6}}$ in the form $p + q\sqrt{6}$
where p and q are rational numbers.

Multiply top and bottom by $(3 + \sqrt{6})$:
Numerator: $(2 + \sqrt{6})(3 + \sqrt{6}) = 12 + 5\sqrt{6}$
Denominator: $9 - 6 = 3$
$= \frac{12 + 5\sqrt{6}}{3}$

Answer: $\mathbf{\frac{12 + 5\sqrt{6}}{3}}$

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5 Similar triangles with algebra

In the diagram, DE is parallel to BC.
AD = x cm, DB = (x + 2) cm, DE = 3 cm and BC = 7 cm.
(a) Find the value of x.
(b) Work out the length of AB.
The diagram is not drawn accurately.

(a) Triangles ADE and ABC are similar, so $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{x}{2x + 2} = \frac{3}{7}$
$7x = 3(2x + 2)$
$7x = 6x + 6$
$x = 6$
(b) AB = 2x + 2 = 14 cm
Answer: (a) x = 6 (b) 14 cm

6 Tree diagrams without replacement

A bag contains 2 red cubes, 3 blue cubes and 2 green cubes.
Two cubes are taken at random without replacement.
Work out the probability that (a) they are different colours, (b) both are the same colour.

(a) P(different) = 1 − P(same) = $1 - \frac{5}{21} = \frac{16}{21}$

(b) Add the ways: $\frac{2}{7}\times\frac{1}{6} + \frac{3}{7}\times\frac{2}{6} + \frac{2}{7}\times\frac{1}{6} = \frac{5}{21}$

Answer: (a) $\frac{16}{21}$ (b) $\frac{5}{21}$

7 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 69°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 69°
Angle COB is an exterior angle of triangle OAC: 69 + 69 = 138°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 138°

8 Bounds of speed, density and pressure

A metal block has a mass of 180 g, correct to 2 significant figures.
Its volume is 13 cm^3, correct to the nearest whole number.
Work out the lower bound of the density.
Give your answer correct to 3 significant figures. (Calculator)

175 ≤ mass < 185; 12.5 ≤ volume < 13.5
Lower bound of density = smallest mass ÷ biggest volume = 175 ÷ 13.5
= 12.9630... = 13.0 (3 s.f.)
Answer: 13.0 g/cm^3

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9 Functions that are their own inverse

$f(x) = \frac{3x - 6}{2x - 3}$ where $x \neq \frac{3}{2}$

(a) Show that ff(x) = x.

(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by $(2x - 3)$

$ff(x) = \frac{3(3x - 6) + (-6)(2x - 3)}{2(3x - 6) - 3(2x - 3)}$

Top $= 9x - 18 - 12x + 18 = -3x$

Bottom $= 6x - 12 - 6x + 9 = -3$

$ff(x) = \frac{-3x}{-3} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{3x - 6}{2x - 3}$

Answer: (b) $\mathbf{f^{-1}(x) = \frac{3x - 6}{2x - 3}}$

10 Histograms with an unmarked scale

The histogram shows the lengths of leaves collected from a tree.
100 of them had a length between 60 and 80 cm.
(a) How many had a length between 40 and 60 cm?
(b) How many leaves are there in total?

Frequency is proportional to the area of a bar
60 to 80: bar is 5 squares high and 20 wide, area 100, which represents 100
So 1 unit of area represents 100 ÷ 100 = 1
(a) 40 to 60: 3 × 20 = 60 units of area, so 60 × 1 = 60
(b) Total area = 80 + 160 + 10 + 60 + 100 = 410, so total = 410 × 1 = 410
Answer: (a) 60 (b) 410