Day 101 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

The interior angles of a hexagon are
$x°$, $(x + 40)°$, $(x + 35)°$, $(x + 20)°$, $(x + 30)°$, $(x + 25)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(6 - 2) \times 180 = 720°$
$6x + 150 = 720$, so $6x = 570$
$x = 95$
(b) The angles are 95°, 135°, 130°, 115°, 125°, 120°
The largest is 135°
Answer: (a) x = 95 (b) 135°

2 Substitution

Work out the value of $xyz - x^2$
when $x = -5$, $y = 1$ and $z = -2$.

Substitute the values: $(-5) \times 1 \times (-2) - (-5)^2$
$= 10 - 25$
$= -15$
Answer: −15

3 Exchange rates

Maya changes £400 into euros at £1 = €1.25.
Maya spends €310 and then changes the rest back into pounds at £1 = €1.6.
How many pounds does Maya get back?

£400 × 1.25 = €500
500 − 310 = €190
190 ÷ 1.6 = £118.75
Answer: £118.75

4 Simplifying algebraic fractions

Simplify fully $\frac{4x^2 - 36}{x^2 - 10x + 21}$

Factorise the top: $4(x - 3)(x + 3)$
Factorise the bottom: $(x - 3)(x - 7)$
Cancel the common factor $(x - 3)$: $\frac{4(x + 3)}{x - 7}$

Answer: $\mathbf{\frac{4(x + 3)}{x - 7}}$

5 Ratio and algebra

In a bag, the ratio of the number of red counters to the number of blue counters is 2 : 7.
Then 17 red counters are removed and 17 blue counters are removed.
The ratio is now 1 : 12.
How many blue counters were there at the start?

Let the numbers at the start be 2k and 7k
$(2k - 17) : (7k - 17) = 1 : 12$, so $12(2k - 17) = 1(7k - 17)$
$24k - 204 = 7k - 17$
$17k = 187$, so $k = 11$
Number of blue counters at the start: $7 \times 11 = 77$
Answer: 77

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6 Perpendicular lines

The diagram shows the straight line L, which passes through the points A and B.
The line M is perpendicular to L and passes through B.
Find the equation of M in the form y = mx + c.

Gradient of L = $\frac{2-0}{0-(-6)} = \frac{1}{3}$

Perpendicular gradient = $-1 \div \frac{1}{3} = -3$

M passes through B(−6, 0): $0 = (-3) \times (-6) + c$, so $c = -18$
$y = -3x - 18$
Answer: $y = -3x - 18$

7 Bounds of area and perimeter

A rectangle has length 12 cm and width 5 cm.
Both measurements are correct to the nearest centimetre.
Amir says that the area of the rectangle could be exactly 73 cm^2.
Is Amir correct? You must show your working.

$11.5 \leq$ length < 12.5 and $4.5 \leq$ width < 5.5
Lower bound of area $= 11.5 \times 4.5 = 51.75$
Upper bound of area $= 12.5 \times 5.5 = 68.75$
So $51.75 \leq$ area < 68.75
73 is outside this interval, so Amir is not correct
Answer: No, 73 is outside the interval

8 Recasting solids

A solid metal sphere of radius 3 cm is melted down and recast into a solid cylinder of radius 2 cm.
Work out the height of the cylinder.

The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Volume of sphere $= \frac{4}{3} \times \pi \times 3^3 = 36\pi$

Volume of cylinder $= \pi \times 2^2 \times h = 4\pi h$
$4h = 36$
$h = 9$ cm
Answer: 9 cm

9 Joint and inverse variation

P is directly proportional to v and inversely proportional to t.
When v = 4 and t = 1, P = 24.
Work out the value of P when v = 2 and t = 2.

$P = \frac{kv}{t}$

$24 = \frac{k \times 4}{1}$, so $k = 24 \times 1 \div 4 = 6$

$P = \frac{6 \times 2}{2} = 6$

Answer: P = 6

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10 Bounds of roots

The area of a circle is 305 cm^2, correct to the nearest cm^2.
Work out the upper bound of the radius of the circle.
Give your answer correct to 3 significant figures. (Calculator)

$304.5 \leq \text{area} < 305.5$

Area = πr^2, so r = $\sqrt{\frac{\text{area}}{\pi}}$

Upper bound = $\sqrt{\frac{305.5}{\pi}}$ = 9.86122... = 9.86 (3 s.f.)

Answer: 9.86 cm