Day 100 **S Tier 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 11:05 (London time).
It takes 12 hours 35 minutes.
When it is 12:00 in London it is 21:00 in Sydney.
At what time, in Sydney time, does the flight land? Say if it is on a different day.

Arrival in London time = 11:05 + 12 h 35 min = 23:40
Sydney is 9 hours ahead of London, so add 9 hours
08:40 the next day
Answer: 08:40 the next day

2 Compound interest

Amir invests £8,000 for 3 years at 4% per year compound interest.
Work out the value of the investment at the end of the 3 years.
Give your answer to the nearest penny. (Calculator)

Multiplier = 1 + 4 ÷ 100 = 1.04
$8000 \times 1.04^3 = 8998.91$
To the nearest penny: £8,998.91
Answer: £8,998.91

3 Simplifying surds

Write $\sqrt{80} - \sqrt{45} + \sqrt{20}$ in the form $a\sqrt{5}$
where a is an integer.

$\sqrt{80} = 4\sqrt{5}$; $\sqrt{45} = 3\sqrt{5}$; $\sqrt{20} = 2\sqrt{5}$
$(4 - 3 + 2)\sqrt{5} = 3\sqrt{5}$
Answer: $3\sqrt{5}$

4 Equations with indices

Solve $3^{2x} = \frac{1}{9}$

Write the right-hand side as a power of 3: $\frac{1}{9} = 3^{-2}$

Equate the powers: $2x = -2$
$2x = -2$
$x = -1$
Answer: $x = -1$

5 Fractions removed from a ratio

The ratio of blue to green sweets in a tin is 7 : 9.
Then $\frac{2}{5}$ of the blue and $\frac{1}{3}$ of the green are taken out.
Work out the ratio of blue to green now.

Left: $\frac{3}{5} \times 7 = \frac{21}{5}$ of the blue part, $\frac{2}{3} \times 9 = 6$ of the green part

$\frac{21}{5} : 6$
Multiply by 5: 21 : 30 = 7 : 10
Answer: 7 : 10

6 Fractional and negative indices

Work out $16^{\frac{5}{4}}$

The denominator 4 means the fourth root: the fourth root of 16 is 2
The numerator 5 means raise to the power 5: $(2)^5 = 32$
Answer: 32

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7 Combined transformations

The point P(6, 3) is reflected in the y-axis.
The image is then rotated 90° clockwise about the origin.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (−6, 3)
After the second transformation: P'' = (3, 6)
(b) Follow a general point (x, y) through both transformations: (x, y) → (−x, y) → (y, x)
The single mapping is (x, y) → (y, x)
This is a reflection in the line y = x
Answer: (a) (3, 6) (b) Reflection in the line y = x

8 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $4\mathbf{a} + 5\mathbf{b}$, OQ = $-2\mathbf{a} + 20\mathbf{b}$ and OR = $-6\mathbf{a} + 30\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $-6\mathbf{a} + 15\mathbf{b}$
QR = OR − OQ = $-4\mathbf{a} + 10\mathbf{b}$
QR = $\frac{2}{3}$ PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 2
Answer: (b) 3 : 2

9 Factorising fully

Factorise fully $4x^3 - 16x$

Take out the common factor 4x: $4x(x^2 - 4)$
$x^2 - 4$ is a difference of two squares: $(x - 2)(x + 2)$
Answer: $4x(x - 2)(x + 2)$

10 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 9 cm, top radius 4 cm and base radius 8 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+9} = \frac{4}{8}$
8 h = 4(h + 9), so 4 h = 36 and h = 9
Large cone: $\frac{1}{3} \times \pi \times 8^2 \times 18 = 384\pi$
Small cone: $\frac{1}{3} \times \pi \times 4^2 \times 9 = 48\pi$
Frustum = $384\pi - 48\pi = 336\pi$ cm^3
Answer: 336π cm^3