Day 10 **S Tier 10-a-day: Model Solutions**

1 Compound interest

Carlos invests £4,500 for 8 years at 6% per year compound interest.
Work out the value of the investment at the end of the 8 years.
Give your answer to the nearest penny. (Calculator)

Multiplier = 1 + 6 ÷ 100 = 1.06
$4500 \times 1.06^8 = 7172.32$
To the nearest penny: £7,172.32
Answer: £7,172.32

2 Ordering numbers

Write these numbers in order of size, starting with the smallest.

32%, 83%, $\frac{3}{4}$, 54%

Convert each to a decimal: 32% = 0.32, 83% = 0.83, $\frac{3}{4}$ = 0.75, 54% = 0.54

In order: 32%, 54%, $\frac{3}{4}$, 83%

Answer: 32%, 54%, $\frac{3}{4}$, 83%

3 Quadratic graphs

The curve $y = -x^2 + 12x - 11$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the maximum point of the curve.

(a) Put y = 0. $-x^2 + 12x - 11 = 0$, so $-(x^2 + 12x + 11) = 0$, giving $-(x - 1)(x - 11) = 0$
$x - 1 = 0$ or $x - 11 = 0$, so x = 1 or x = 11
(b) The turning point is halfway between A and B: x = (1 + 11) ÷ 2 = 6
When x = 6: $y = -1 \times 6^2 + 12 \times 6 - 11 = 25$
Answer: (a) A(1, 0), B(11, 0) (b) (6, 25)

4 Solving trigonometric equations

Solve $2 \sin x + 1 = 0$ for $0° \leq x \leq 360°$. (Calculator)
Give your answers correct to 1 decimal place.

$\sin x = -\frac{1}{2}$

Related acute angle: $\sin^{-1}(\frac{1}{2}) = 30.0°$

sine is negative in the 3rd and 4th quadrants, so $x = 180° + 30.0°$ and $x = 360° - 30.0°$
x = 210.0° and x = 330.0°
Answer: x = 210.0° or x = 330.0°

5 Upper and lower bounds

a = 35 and b = 13,
both correct to the nearest whole number.
Work out the lower bound of a + b.

Each value is rounded, so it can be up to 0.5 above or below
$34.5 \leq a < 35.5$; $12.5 \leq b < 13.5$
Lower bound = 34.5 + 12.5 = 47
Answer: 47

6 Transformations of graphs

The point P(−4, 1) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of
(a) y = f(x − 3) (b) y = 2f(x)

(a) y = f(x − 3) is a translation of 3 to the right: (−1, 1)
(b) y = 2f(x) is a vertical stretch with scale factor 2, so the y coordinate is doubled: (−4, 2)
Answer: (a) (−1, 1) (b) (−4, 2)

7 Angles of elevation

A point A is on level ground in line with the base of a cliff. (Calculator)
The angle of elevation of the top of the cliff from A is 51°.
B is a point on the same line, 100 m further from the cliff than A.
The angle of elevation of the top of the cliff from B is 34°.
Work out the height of the cliff. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 51° = \frac{h}{x}$, so $x = h \div \tan 51°$

From B: $\tan 34° = \frac{h}{x + 100}$, so $x + 100 = h \div \tan 34°$

Subtract: $100 = h \div \tan 34° - h \div \tan 51° = h(1.4826 - 0.80978)$
$h = 100 \div 0.67278 = 148.6$ m (1 d.p.)
Answer: 148.6 m

8 Ratio and algebra

In an animal shelter, the ratio of the number of cats to the number of dogs is 3 : 7.
Then 17 cats are removed and 8 more dogs are added.
The ratio is now 1 : 6.
How many cats were there at the start?

Let the numbers at the start be 3k and 7k
(3k − 17) : (7k + 8) = 1 : 6, so 6(3k − 17) = 1(7k + 8)
18k − 102 = 7k + 8
11k = 110, so k = 10
Number of cats at the start: 3 × 10 = 30
Answer: 30

9 Bounds of roots

The volume of a cube is 655 cm^3, correct to the nearest cm^3.
Work out the lower bound of the edge length of the cube.
Give your answer correct to 3 significant figures. (Calculator)

$654.5 \leq$ volume < 655.5
Lower bound of edge $= \sqrt[3]{654.5}$
$= 8.68234... = 8.68$ (3 s.f.)
Answer: 8.68 cm

10 Rearranging when the subject appears twice

Make t the subject of $T = \frac{4t + 6}{2t - 4}$

Multiply by 2t − 4: $T(2t - 4) = 4t + 6$
Expand: $2tT - 4T = 4t + 6$
Collect the t terms: $2tT - 4t = 6 + 4T$
Factorise: $t(2T - 4) = 4T + 6$
$t = \frac{4T + 6}{2T - 4}$

Answer: $t = \frac{4T + 6}{2T - 4}$