Day 99 **IGCSE 10-a-day: Model Solutions**

1 Local maximum and minimum values

$f(x) = x^3 - 12x$

Find the y coordinate of the local maximum point and the y coordinate of the local minimum point.

$f'(x) = 3x^2 - 12 = 3(x - 2)(x + 2)$

$f'(x) = 0$ gives $x = -2$ or $x = 2$

$f''(x) = 6x$: $f''(-2) = -12 < 0$ (maximum) and $f''(2) = 12 > 0$ (minimum)

$f(-2) = 16$ and $f(2) = -16$

Answer: Local maximum y = 16, local minimum y = −16

2 Vector between two points

A is the point (0, 1) and B is the point (−1, −4).

(a) Write down the vector **AB**.

(b) Find |**AB**|, giving the answer as a surd in simplest form if it is not exact.

(a) $\mathbf{AB} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -5 \end{pmatrix}$

(b) $|\mathbf{AB}| = \sqrt{(-1)^2 + (-5)^2} = \sqrt{26}$

Answer: (a) $\begin{pmatrix} \mathbf{-1} \\ \mathbf{-5} \end{pmatrix}$ **(b)** $\sqrt{\mathbf{26}}$

3 Gradient with fractional or negative powers

The equation of a curve is $y = -x^2 - \frac{12}{x}$.

Work out the gradient of the curve at the point where x = 1.

$\frac{dy}{dx} = -2x + 12x^{-2}$

When x = 1: gradient = 10

Answer: 10

4 Enlargement

Triangle P has vertices A(0, 4), B(−2, 6) and C(2, 1).

Triangle Q has vertices A'(8, 4), B'(14, −2) and C'(2, 13).

Q is an enlargement of P.

Describe fully the single transformation that maps P onto Q.

x-step from A to B = −2, x-step from A' to B' = 6

Scale factor = 6 ÷ −2 = −3

The centre C satisfies: C + −3(A − C) = A', so (1 − −3)C = A' − −3 A

x: 4 × x = 8 − (−3) × 0 = 8, so x = 2

y: 4 × y = 4 − (−3) × 4 = 16, so y = 4

Answer: Enlargement, scale factor −3, centre (2, 4)

5 Sample space and conditional probability

A fair 4-sided dice and a fair 6-sided dice are each thrown once. The first score and the second score are recorded.

(a) Work out the probability that the sum is greater than 5.

(b) Given that the first number is even, work out the probability that the sum is greater than 5.

(a) There are 4 × 6 = 24 equally likely outcomes, and 14 of them satisfy: the sum is greater than 5

Probability = $\frac{7}{12}$

(b) 12 outcomes satisfy: the first number is even. Of these, 8 also satisfy: the sum is greater than 5

Probability = $\frac{2}{3}$

Answer: (a) $\frac{\mathbf{7}}{\mathbf{12}}$ **(b)** $\frac{\mathbf{2}}{\mathbf{3}}$

6 Quadrilateral: cosine rule, then area

In quadrilateral ABCD, AB = 7 cm, BC = 11 cm, CD = 12 cm,
angle ABC = 56° and angle ACD = 50°.
Work out the area of ABCD. Give your answer correct to 3 significant figures. (Calculator)

$AC^2 = 7^2 + 11^2 - 2 \times 7 \times 11 \times \cos 56°$ so AC = 9.1588 cm
Area ABC = ½ × 7 × 11 × sin 56° = 31.92
Area ACD = ½ × 9.1588 × 12 × sin 50° = 42.10
Total area = 31.92 + 42.10 = 74.0

Answer: 74.0 cm^2

7 Stationary point of y = ax + b/√x

A curve has equation $y = x + \frac{16}{\sqrt{x}}$ for x > 0.

Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 1 - 8x^{-3/2}$

$\frac{dy}{dx} = 0$: $8x^{-3/2} = 1$, so $x^{3/2} = 8$

$x = 8^{2/3} = 4$, and y = 12

$\frac{d^2y}{dx^2} = 12x^{-5/2}$

At x = 4, $\frac{d^2y}{dx^2} = \frac{3}{8}$, which is positive, so a minimum

Answer: (4, 12), minimum

8 Profit and loss

A trader buys a watch and adds a 25% mark-up to get the selling price.
In a sale the selling price is reduced by 15% to £297.50.
Work out how much the shop paid for a watch.

Overall multiplier = 1.25 × 0.85 = 1.0625
Cost = £297.50 ÷ 1.0625
= £280

Answer: £280

9 Ratio and expressions

4x = 3y
Work out the ratio (3x + 1y) : (5y − 4x)
Give your answer in its simplest form.

4x = 3y gives x : y = 3 : 4, so let x = 3t and y = 4t
3(3t) + 1(4t) = 13t
5(4t) − 4(3t) = 8t
13t : 8t = 13 : 8

Answer: 13 : 8

10 Equations with indices

Solve $5^{x+1} = 25^{x+1}$

Write both sides as powers of 5: $(5^1)^{x+1} = (5^2)^{x+1}$
$5^{x+1} = 5^{2x+2}$
Equate the powers: x + 1 = 2x + 2
−x = 1
x = −1

Answer: x = −1