Day 98 **IGCSE 10-a-day: Model Solutions**

1 Angles in polygons

Each interior angle of a regular polygon is 11 times the size of each exterior angle.
Work out the number of sides of the polygon.

Let each exterior angle be e. Then each interior angle is 11e.
Interior angle + exterior angle = 180°, so 11e + e = 180°
e = 180° ÷ 12 = 15°
Number of sides = 360° ÷ 15° = 24
Answer: 24

2 Time zones

A flight leaves London at 19:35 (London time).
It takes 10 hours 40 minutes.
When it is 12:00 in London it is 07:00 in New York.
At what time, in New York time, does the flight land? Say if it is on a different day.

Arrival in London time = 19:35 + 10 h 40 min = 06:15 the next day
New York is 5 hours behind London, so subtract 5 hours
01:15 the next day
Answer: 01:15 the next day

3 Tangent at a turning point

The graph of $y = 3x^2 + 24x + 9$ has a turning point at P.
Find the equation of the tangent to the curve at P.

$\frac{dy}{dx} = 6x + 24$

At P, $\frac{dy}{dx} = 0$, so $6x + 24 = 0$ and $x = -4$

$y = -39$ at P
The gradient is 0, so the tangent is horizontal: $y = -39$
Answer: $y = -39$

4 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle APB = 46°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angles in quadrilateral OAPB: 360 − 90 − 90 − 46 = 134°
(b) The angle at the circumference is half the angle at the centre: 134 ÷ 2 = 67°
Answer: (a) 134° (b) 67°

5 Linear programming: vertices

x and y satisfy $x \geq 0$, $y \geq 0$,
$3x + 4y \leq 36$ and $6x + 4y \leq 60$.
Find the greatest value of $P = 5x + 6y$ and the values of x and y that give it.

Draw the lines. The feasible region has vertices (0, 0), (0, 9), (8, 3), (10, 0)
Values of P: (0, 0): 0, (0, 9): 54, (8, 3): 58, (10, 0): 50
Greatest value of P is 58 at (8, 3)
Answer: P = 58 when x = 8 and y = 3

6 Equation of a circle

A circle has centre C(−3, 3) and passes through the point P(5, 7).
Find an equation of the circle.

$r^2 = CP^2 = (5 - (-3))^2 + (7 - 3)^2$
$= 64 + 16 = 80$
$(x + 3)^2 + (y - 3)^2 = 80$
Answer: $(x + 3)^2 + (y - 3)^2 = 80$

7 No stationary points

A curve has equation $y = 2x^3 + 18x^2 + 57x + 2$.
(a) Show that the curve has no stationary points.
(b) Find the least value of the gradient of the curve.

$\frac{dy}{dx} = 6x^2 + 36x + 57 = 6(x + 3)^2 + 3$

A squared term is never negative, so $\frac{dy}{dx} \geq 3 > 0$ for all x

So $\frac{dy}{dx}$ is never zero, and there are no stationary points

The least gradient is 3 (when $x = -3$)
Answer: (b) 3

8 Speed and a moving train

A train travels at a constant speed of 54 km/h.
It takes 40 seconds to go completely through a tunnel which is 270 m long.
Work out the length of the train, in metres.

Speed = 54 km/h = 54,000 ÷ 3600 = 15 m/s
Distance travelled in 40 s = 15 × 40 = 600 m
This is the length of the tunnel plus the length of the train
Train = 600 − 270 = 330 m
Answer: 330 m

9 Open box of fixed surface area

An open container has a square base of side x cm and height h cm. The total outside surface area is 432 cm^2.

(a) Show that the volume of the box is $V = \frac{1}{4}(432x - x^3)$ cm^3.

(b) Find the maximum volume of the box, proving that it is a maximum.

$x^2 + 4xh = 432$, so $h = \frac{432 - x^2}{4x}$

$V = x^2h = \frac{x(432 - x^2)}{4} = \frac{1}{4}(432x - x^3)$

$\frac{dV}{dx} = -\frac{3}{4}x^2 + 108 = 0$, so $x^2 = 144$ and $x = 12$

$\frac{d^2V}{dx^2} = -\frac{3}{2}x = -18 < 0$, so a maximum

$V = 864$
Answer: 864 cm^3

10 Range and inverse of a quadratic

$g(x) = x^2 - 6x + 11$ for $x \leq 3$.

(a) Find the range of g.

(b) Find $g^{-1}(x)$.

(a) $g(x) = (x - 3)^2 + 2$

$x \leq 3$ includes the minimum point, so $g(x) \geq 2$

(b) $y = (x - 3)^2 + 2$, so $(x - 3)^2 = y - 2$

$x \leq 3$ means $x - 3$ is negative or zero, so $x = 3 - \sqrt{y - 2}$

$g^{-1}(x) = 3 - \sqrt{x - 2}$

Answer: (a) $g(x) \geq 2$ (b) $g^{-1}(x) = 3 - \sqrt{x - 2}$