Day 97 **IGCSE 10-a-day: Model Solutions**

1 Gradient from a tangent

The diagram shows part of the curve $y = x^2 - 5x - 2$. The line shown is the tangent to the curve at the point P. (Not drawn accurately.)
(a) Use the two marked points on the tangent to work out its gradient.
(b) Use differentiation to find the x coordinate of P.

Gradient $= \frac{-7 - (-10)}{4 - 1} = 1$

$\frac{dy}{dx} = 2x - 5$

$2x - 5 = 1$, so $x = 3$

Answer: (a) 1 (b) x = 3

2 Error intervals

The height of a tower is 5400 m, correct to the nearest 100.
Write down the error interval for the height of a tower, using h.

The value has been rounded to the nearest 100
Half of this is 50, so go 50 below and 50 above
$5400 - 50 = 5350$ and $5400 + 50 = 5450$
$5350 \leq h < 5450$

Answer: $5350 \leq h < 5450$

3 Comparing costs

Sparkle Cleaners has a call-out fee of £7 plus £3 per hour.
Fresh Start has a call-out fee of £17.50 plus £1.50 per hour.
(a) Work out the number of hours for which the two costs are the same.
(b) Which is cheaper for 16 hours, and by how much?

(a) Let n be the number of hours. Working in pounds, the costs are equal when $7 + 3n = 17.50 + 1.50n$
$1.50n = 10.50$, so $n = 7$
(b) Sparkle Cleaners: £7 + £3 × 16 = £55
Fresh Start: £17.50 + £1.50 × 16 = £41.50
Fresh Start is cheaper by £13.50

Answer: (a) 7 (b) Fresh Start, by £13.50

4 Area scale factor and determinant

Shape S has area 8 cm^2.

S is transformed by the matrix $\begin{pmatrix} k & 3 \\ 3 & 5 \end{pmatrix}$ and the image has area 88 cm^2.

Find the two possible values of k.

Area scale factor = 88 ÷ 8 = 11
Determinant = $5k - 9 = \pm 11$

$5k - 9 = 11$ gives $k = 4$; $5k - 9 = -11$ gives $k = -\frac{2}{5}$

Answer: $k = -\frac{2}{5}$ or $k = 4$

5 Intersection and gradient

The curve $y = x^2 + x + 3$ and the line $y = -4x - 3$ intersect at two points A and B, where A has the smaller x coordinate.
(a) Find the coordinates of A and B.
(b) Find the gradient of the curve at A and at B.

$x^2 + x + 3 = -4x - 3$ gives $x^2 + 5x + 6 = 0$, so $(x + 3)(x + 2) = 0$
$x = -3$ or $x = -2$, giving A $(-3, 9)$ and B $(-2, 5)$

$\frac{dy}{dx} = 2x + 1$: at A it is -5, at B it is -3

Answer: (a) A $(-3, 9)$, B $(-2, 5)$ (b) -5 at A, -3 at B

6 Shaded areas: exact answers

The diagram shows an equilateral triangle inside a circle of radius 6 cm.
The three vertices of the triangle lie on the circle.
Work out the area of the shaded region. Give your answer in the form $a\pi - b\sqrt{3}$.

Split the triangle into 3 isosceles triangles with two sides of 6 cm and an angle of 120° between them

Area of the triangle = $3 \times \frac{1}{2} \times 6 \times 6 \times \sin 120° = 54 \times \frac{\sqrt{3}}{2} = 27\sqrt{3}$

Area of circle = $\pi \times 6^2 = 36\pi$

Shaded area = $36\pi - 27\sqrt{3}$ cm^2

Answer: $(36\pi - 27\sqrt{3})$ cm^2

7 Normal meets the x-axis

The normal to the curve $y = -x^2 - 5x + 2$ at the point where $x = -2$ meets the x-axis at N.
Find the coordinates of N.

y = 8 and gradient of curve = −1
Gradient of normal = 1
y − 8 = x + 2
Put y = 0: −8 = x + 2
x = −10

Answer: (−10, 0)

8 Vector proof: collinear points

OA = **a** and OB = **b**.
OP = 5**a** − 2**b**, OQ = 7**a** + 8**b** and OR = 10**a** + 23**b**.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = 2**a** + 10**b**
QR = OR − OQ = 3**a** + 15**b**

QR = $\frac{3}{2}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line
(b) PQ : QR = 2 : 3

Answer: (b) 2 : 3

9 Bounds of a compound expression

p = 11.1 and r = 8.5, each correct to 1 decimal place.
q = 3.15, correct to 2 decimal places.

Work out the lower bound of $\frac{p + q}{r}$. Give your answer correct to 3 significant figures. (Calculator)

For the lower bound use LB of the top and UB of the bottom
Top: LB p + LB q = 11.05 + 3.145 = 14.195
Bottom: UB r = 8.55
14.195 ÷ 8.55 = 1.66

Answer: 1.66

10 Reverse percentages

The price of a tablet is increased by 40%.
Later the new price is reduced by 15% in a sale.
The sale price is £130.90.
Work out the price before the increase. (Calculator)

Multipliers: 1.4 and 0.85
Overall multiplier = 1.4 × 0.85 = 1.19
Original × 1.19 = £130.90
Original = £130.90 ÷ 1.19 = £110

Answer: £110