Day 96 **IGCSE 10-a-day: Model Solutions**

1 Reverse percentages

The price of a tablet is increased by 20%.
Later the new price is reduced by 20% in a sale.
The sale price is £460.80.
Work out the price before the increase. (Calculator)

Multipliers: 1.2 and 0.8
Overall multiplier = 1.2 × 0.8 = 0.96
Original × 0.96 = £460.80
Original = £460.80 ÷ 0.96 = £480
Answer: £480

2 Points with a given gradient (cubic)

The curve $y = 2x^3 - 13x - 7$ has gradient 11 at two points.
Find the coordinates of these points.

$\frac{dy}{dx} = 6x^2 - 13$
$6x^2 - 13 = 11$, so $6x^2 - 24 = 0$
Divide by 6: $x^2 - 4 = 0$
$(x + 2)(x - 2) = 0$, so $x = -2$ or $x = 2$
Substitute into y: $(-2, 3)$ and $(2, -17)$
Answer: $(-2, 3)$ and $(2, -17)$

3 Solving quadratics by factorising

Solve $3x^2 - 19x + 28 = 0$

Factorise: $(3x - 7)(x - 4) = 0$
$3x - 7 = 0$ or $x - 4 = 0$
$x = \frac{7}{3}$ or $x = 4$
Answer: $x = \frac{7}{3}$ or $x = 4$

4 Solve a composite function equation

$f(x) = -3x - 3$ and $g(x) = x^2 - 9$.
Solve $gf(x) = -8$.

$gf(x) = (-3x - 3)^2 - 9$
$(-3x - 3)^2 = 1$, so $-3x - 3 = \pm 1$
$-3x - 3 = 1$ gives $x = -\frac{4}{3}$; $-3x - 3 = -1$ gives $x = -\frac{2}{3}$
Answer: $x = -\frac{4}{3}$ or $x = -\frac{2}{3}$

5 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 3 cm, DB = 9 cm and the area of triangle ADE is 4 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 3 + 9 = 12 cm
Length scale factor = 12 ÷ 3 = 4
Area scale factor = $(4)^2 = 16$
Area of triangle ABC = 4 × 16 = 64 cm^2
Trapezium = 64 − 4 = 60 cm^2
Answer: 60 cm^2

6 Intersection and gradient

The curve $y = 2x^2 - 5x - 4$ and the line $y = -3x$ intersect at two points A and B, where A has the smaller x coordinate.
(a) Find the coordinates of A and B.
(b) Find the gradient of the curve at A and at B.

$2x^2 - 5x - 4 = -3x$ gives $2x^2 - 2x - 4 = 0$, so $2(x + 1)(x - 2) = 0$
$x = -1$ or $x = 2$, giving A $(-1, 3)$ and B $(2, -6)$
$\frac{dy}{dx} = 4x - 5$: at A it is -9, at B it is 3
Answer: (a) A $(-1, 3)$, B $(2, -6)$ (b) -9 at A, 3 at B

7 Percentages of percentages

In a group of students, 55% are boys and the rest are girls.
35% of the boys and 25% of the girls walk to school.
What percentage of all the students walk to school?

Take 100 students: 55 are boys and 45 are girls
35% of 55 = 19.25 and 25% of 45 = 11.25
19.25 + 11.25 = 30.5 out of 100
Answer: 30.5%

8 Bounds: finding an unknown length

A rectangle has area 82 cm^2, correct to 2 significant figures.
Its length is 14 cm, correct to 2 significant figures.
Work out the upper bound of the width of the rectangle. Give your answer correct to 3 significant figures. (Calculator)

Width = area ÷ length
Area: $81.5 \leq A < 82.5$. Length: $13.5 \leq l < 14.5$
UB width = UB area ÷ LB length = 82.5 ÷ 13.5 = 6.11
Answer: 6.11 cm

9 Tangent meets the curve again (cubic)

The tangent to the curve $y = x^3 - 4x^2 + 5x - 7$ at the point P where $x = 1$ meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 3x^2 - 8x + 5$, so at P the gradient is 0 and $y = -5$
Tangent: $y = -5$
Intersection: $x^3 - 4x^2 + 5x - 2 = 0$
$x = 1$ is a repeated root, so the roots are 1, 1 and x_Q
Sum of roots = 4, so $2 + x_Q = 4$ and $x_Q = 2$
$y = -5$
Answer: $(2, -5)$

10 Perpendicular bisector

A is the point $(-3, 4)$ and B is the point $(5, -2)$.
Find an equation of the perpendicular bisector of AB.
Give your answer in the form $y = mx + c$.

Midpoint of AB = $(\frac{-3 + 5}{2}, \frac{4 + (-2)}{2}) = (1, 1)$

Gradient of AB = $\frac{-2 - 4}{5 - (-3)} = -\frac{3}{4}$

The perpendicular gradient is $-1 \div (-\frac{3}{4}) = \frac{4}{3}$

Using the midpoint: $y - 1 = \frac{4}{3}(x - 1)$

$y = \frac{4}{3}x - \frac{1}{3}$

Answer: $y = \frac{4}{3}x - \frac{1}{3}$