Day 95 **IGCSE 10-a-day: Model Solutions**

1 Differentiating polynomials

Differentiate $4x^5 - 8x^4 + 2x$ with respect to x.

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$

$4x^5 \to 20x^4$, $-8x^4 \to -32x^3$, $2x \to 2$

$\frac{dy}{dx} = 20x^4 - 32x^3 + 2$

Answer: $\mathbf{\frac{dy}{dx} = 20x^4 - 32x^3 + 2}$

2 Venn diagram: reading set notation

The Venn diagram shows information about 15 pupils.
A: walk to school B: cycle to school
(a) Find n(A′).
(b) Find n(B′).

(a) n(A′) = 3 + 1 = 4
(b) n(B′) = 3 + 1 = 4
Answer: (a) 4 (b) 4

3 Gradient at a point

Work out the gradient of the curve $y = x^4 - 8x^2 - 6x$ at the point where x = 0.

$\frac{dy}{dx} = 4x^3 - 16x - 6$
When x = 0: gradient = −6
Answer: −6

4 Overall percentage change from repeated changes

A population grows by 20% each year for 4 years.
Work out the overall percentage increase. Give your answer correct to 3 significant figures.

Multiplier = 1 + 20 ÷ 100 = 1.2
Overall multiplier = $1.2^4 = 2.0736$
Percentage increase = (2.0736 − 1) × 100 = 107%
Answer: 107%

5 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 9 cm, DB = 6 cm and the area of triangle ADE is 144 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 9 + 6 = 15 cm
Length scale factor = $15 \div 9 = \frac{5}{3}$
Area scale factor = $(\frac{5}{3})^2 = \frac{25}{9}$
Area of triangle ABC = $144 \times \frac{25}{9} = 400$ cm^2
Trapezium = 400 − 144 = 256 cm^2
Answer: 256 cm^2

6 Expanding brackets with surds

Expand and simplify $(1 + 2\sqrt{5})^2$
Give your answer in the form $p + q\sqrt{5}$

$(1 + 2\sqrt{5})^2 = 1 + 4\sqrt{5} + 20$
$= 21 + 4\sqrt{5}$
Answer: $21 + 4\sqrt{5}$

7 Factorising fully

Factorise fully $(x + 3)^2 - 64$

This is a difference of two squares: $(x + 3)^2 - 8^2$
$= (x + 3 - 8)(x + 3 + 8)$
$= (x - 5)(x + 11)$
Answer: $(x - 5)(x + 11)$

8 Solve f′(x) = 0 with a reciprocal term

$f(x) = 4x^3 + \frac{3072}{x}$

Find the values of x for which $f'(x) = 0$.

$f'(x) = 12x^2 - \frac{3072}{x^2}$

$f'(x) = 0$: $12x^2 = \frac{3072}{x^2}$

$x^4 = 256$
$x = 4$ or $x = -4$
Answer: $x = 4$ or $x = -4$

9 Estimating the median from grouped data

The table gives information about 120 times.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 24, 43, 54, 84, 97, 120
The median is the value at position $120 \div 2 = 60$
It is in the class 40 to 50, with 54 values before it, so we need 6 more of the 30 values in this class

Median $\approx 40 + \frac{6}{30} \times 10 = 42$

Answer: 42

10 Bounds and accuracy

a = 0.564 and b = 0.544, both correct to 3 significant figures.
Work out $a \div b$.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

$0.5635 \leq a < 0.5645$; $0.5435 \leq b < 0.5445$
Lower bound = $0.5635 \div 0.5445 = 1.0349...$
Upper bound = $0.5645 \div 0.5435 = 1.0386...$
Both bounds round to 1.0 to 2 significant figures, but not to 3 significant figures
Answer: 1.0 (2 s.f.)