Day 94 **IGCSE 10-a-day: Model Solutions**

1 Tangent at a turning point

The graph of $y = x^2 + 2x + 3$ has a turning point at P.
Find the equation of the tangent to the curve at P.

$\frac{dy}{dx} = 2x + 2$

At P, $\frac{dy}{dx} = 0$, so $2x + 2 = 0$ and $x = -1$

$y = 2$ at P

The gradient is 0, so the tangent is horizontal: $y = 2$

Answer: $y = 2$

2 Exchange rates

Nadia changes £900 into dollars at £1 = $1.2.
Nadia spends $390 and then changes the rest back into pounds at £1 = $1.25.
How many pounds does Nadia get back?

£900 × 1.2 = $1080
1080 − 390 = $690
690 ÷ 1.25 = £552

Answer: £552

3 HCF and LCM

$A = 2^3 \times 3 \times 5^2$
$B = 2^4 \times 3^2 \times 5$
Write down, as a product of prime factors,
(a) the HCF of A and B
(b) the LCM of A and B

HCF: take the lowest power of each prime that appears in both numbers
$= 2^3 \times 3 \times 5$
LCM: take the highest power of every prime that appears
$= 2^4 \times 3^2 \times 5^2$

Answer: (a) $2^3 \times 3 \times 5$ (b) $2^4 \times 3^2 \times 5^2$

4 Exponential growth and decay

A population of fish starts at 400 and grows by 50% each year.
After how many whole years will the population first be at least double its starting value? (Calculator)

Multiplier = 1.5, double is 800
After 1 years: $400 \times 1.5^1 = 600.0$
After 2 years: $400 \times 1.5^2 = 900.0$

Answer: 2 years

5 Rate of cooling or growth

The temperature, T °C, of a cup of tea t minutes after it is removed from a heat source is modelled by

$T = 25 + \frac{80}{t}$ $(t \geq 1)$

(a) Find $\frac{dT}{dt}$ when $t = 2$.

(b) Find the value of t when the temperature is falling at 5 °C per minute.

$T = 25 + 80t^{-1}$, so $\frac{dT}{dt} = -80t^{-2}$

When $t = 2$: $\frac{dT}{dt} = -20$

$-\frac{80}{t^2} = -5$, so $t^2 = 16$ and $t = 4$

Answer: (a) −20 °C/min (b) $t = 4$

6 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 35° and angle CAQ = 46°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 46° (alternate segment theorem)
(b) Angle ACB = angle PAB = 35° (alternate segment theorem)
Answer: (a) 46° (b) 35°

7 Area of a triangle given three sides

Work out the area of triangle ABC.
Give your answer correct to 3 significant figures. (Calculator)

$\cos A = \frac{9^2 + 13^2 - 10^2}{2 \times 9 \times 13} = \frac{25}{39}$

A = 50.132°
Area = ½ × 9 × 13 × sin 50.132° = 44.9
Answer: 44.9 cm^2

8 Cylinder of fixed surface area

A closed cylinder (a drum) has radius r cm, height h cm and total surface area 96π cm^2.
(a) Show that its volume, V cm^3, is V = 48πr − πr^3.
(b) Find the maximum value of V in terms of π, and show that h = 2r when V is a maximum.

$2\pi r^2 + 2\pi rh = 96\pi$, so $h = \frac{48 - r^2}{r}$

$V = \pi r^2 h = \pi r(48 - r^2) = 48\pi r - \pi r^3$

$\frac{dV}{dr} = 48\pi - 3\pi r^2 = 0$, so $r^2 = 16$ and $r = 4$

$\frac{d^2V}{dr^2} = -6\pi r < 0$, so a maximum. $h = \frac{48 - 16}{4} = 8 = 2r$

$V = \pi \times 4^2 \times 8 = 128\pi$
Answer: 128π cm^3

9 nth term of a cubic sequence

Here are the first five terms of a sequence.
−5, 0, 17, 52, 111
The nth term is of the form $an^3 + bn + c$.
Find the values of a, b and c.

1st differences: 5, 17, 35, 59. 2nd: 12, 18, 24. 3rd: 6, 6
Third differences = 6a = 6 so a = 1
Subtract n^3: −6, −8, −10, −12, −14
This is linear with common difference −2, so b = −2, and c = −6 − (−2) = −4
Answer: a = 1, b = −2, c = −4

10 Composite functions: fg and gf

f(x) = −3x − 5 and $g(x) = x^2 - 3$
Solve fg(x) = gf(x)

$fg(x) = f(x^2 - 3) = -3x^2 + 4$
$gf(x) = g(-3x - 5) = (-3x - 5)^2 - 3 = 9x^2 + 30x + 22$
Set them equal and rearrange: $12x^2 + 30x + 18 = 0$
Divide by 6 and factorise: (x + 1)(2x + 3) = 0

$x = -\frac{3}{2}$ or x = −1

Answer: $x = -\frac{3}{2}$ or x = −1