Day 92 **IGCSE 10-a-day: Model Solutions**

1 No stationary points

A curve has equation $y = x^3 + 3x^2 + 10x + 5$.
(a) Show that the curve has no stationary points.
(b) Find the least value of the gradient of the curve.

$\frac{dy}{dx} = 3x^2 + 6x + 10 = 3(x + 1)^2 + 7$

A squared term is never negative, so $\frac{dy}{dx} \geq 7 > 0$ for all x

So $\frac{dy}{dx}$ is never zero, and there are no stationary points

The least gradient is 7 (when $x = -1$)

Answer: (b) 7

2 Time zones

A flight leaves London at 13:45 (London time).
It takes 8 hours 55 minutes.
When it is 12:00 in London it is 17:00 in Delhi.
At what time, in Delhi time, does the flight land? Say if it is on a different day.

Arrival in London time = 13:45 + 8 h 55 min = 22:40
Delhi is 5 hours ahead of London, so add 5 hours
03:40 the next day

Answer: 03:40 the next day

3 Inclusion and exclusion with two sets

There are 49 customers.
18 of them ordered tea and 13 of them ordered coffee.
23 of them do neither.
(a) Work out how many do both.
(b) One of the customers is chosen at random. Find the probability that they only ordered tea.

Number in A or B = 49 − 23 = 26
(a) $n(A \cap B) = n(A) + n(B) - n(A \cup B) = 18 + 13 - 26 = 5$
(b) Number who only ordered tea = 18 − 5 = 13

Probability = $\frac{13}{49}$

Answer: (a) 5 (b) $\frac{13}{49}$

4 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 66° and angle CAQ = 73°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 73° (alternate segment theorem)
(b) Angle ACB = angle PAB = 66° (alternate segment theorem)

Answer: (a) 73° (b) 66°

5 Points satisfying inequalities

R is the region that satisfies all of these inequalities:
$y \leq -x + 3$ $x + y < 9$ $x > 0$
Which of these points lie in R?
A(1, 0) B(2, 7) C(4, 4) D(6, 7) E(8, 3)

A(1, 0): $y \leq 2$: yes; $x + y = 1$: yes; $x = 1$: yes
B(2, 7): $y \leq 1$: no; $x + y = 9$: no; $x = 2$: yes
C(4, 4): $y \leq -1$: no; $x + y = 8$: yes; $x = 4$: yes
D(6, 7): $y \leq -3$: no; $x + y = 13$: no; $x = 6$: yes
E(8, 3): $y \leq -5$: no; $x + y = 11$: no; $x = 8$: yes
In R: A
Answer: A

6 At least one

The probability that the 8:15 train is late on any day is $\frac{1}{4}$.

Events on different occasions are independent.
Work out the probability that this happens at least once in 6 days.
Give your answer correct to 3 significant figures. (Calculator)

$P(\text{never}) = (1 - \frac{1}{4})^6 = \frac{3}{4}^6 = 0.177978515625$

$P(\text{at least once}) = 1 - P(\text{never}) = 1 - 0.177978515625$
$= 0.822$
Answer: 0.822

7 Fencing with a partition

A rectangular pen is x metres wide and y metres long. It is fenced all round and divided into three equal parts by two fences each of length x metres, parallel to the width.
The total length of fencing used is 96 m.
(a) Show that the area of the pen is $A = -2x^2 + 48x$.
(b) Find the maximum possible area, justifying that it is a maximum.

Fencing: $4x + 2y = 96$, so $y = \frac{96 - 4x}{2}$

$A = xy = \frac{x(96 - 4x)}{2} = -2x^2 + 48x$

$\frac{dA}{dx} = -4x + 48 = 0$ gives $x = 12$

$\frac{d^2A}{dx^2} = -4 < 0$, so a maximum

$A = 288$
Answer: 288 m²

8 Trigonometric equations

Solve $4 \sin x° + 6 = 6$ for $0 \leq x \leq 360$

Rearrange: $4 \sin x° = 0$, so $\sin x° = 0$
From the graph of $y = \sin x°$, $\sin x° = 0$ when $x = 0$ or 180 or 360
Answer: x = 0, 180, 360

9 Increasing range for y = ax + b/x

$y = 2x + \frac{50}{x}$

Find the values of x for which y is increasing.

$\frac{dy}{dx} = 2 - \frac{50}{x^2}$

$\frac{dy}{dx} > 0$: $2 > \frac{50}{x^2}$

$x^2 > 25$ (x^2 is positive, so multiply through)

$x < -5$ or $x > 5$

Answer: $x < -5$ or $x > 5$

10 HCF and LCM

The HCF of two numbers is 2 and their LCM is 48.
One of the numbers is 16.
Work out the other number.

HCF × LCM = product of the two numbers

2 × 48 = 96

Other number = 96 ÷ 16 = 6

Answer: 6