Day 91 **IGCSE 10-a-day: Model Solutions**

1 Conditional probability from a two-way table

The table shows how 119 adults choose a drink.

Two people are picked at random without replacement. Work out the probability that both are in the group "Women" and chose Coffee.

First: $\frac{21}{119}$

Second: $\frac{20}{118}$

$\frac{21}{119} \times \frac{20}{118} = \frac{30}{1003}$

Answer: $\mathbf{\frac{30}{1003}}$

2 Expand then differentiate

$y = (x - 3)(x + 1)$

Find $\frac{dy}{dx}$.

Expand: $y = x^2 - 2x - 3$

$x^2 \to 2x$, $-2x \to -2$

$\frac{dy}{dx} = 2x - 2$

Answer: $\mathbf{\frac{dy}{dx} = 2x - 2}$

3 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle ACB = 41°.

(a) Work out the size of angle AOB.

(b) Work out the size of angle OAB.

The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: $2 \times 41 = 82°$

(b) OA = OB (radii), so triangle OAB is isosceles

Angle OAB = $(180 - 82) \div 2 = 49°$

Answer: (a) 82° (b) 49°

4 Rationalising the denominator

Rationalise the denominator of $\frac{6}{2\sqrt{3}}$

Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{3}$:

$\frac{6}{2\sqrt{3}} = \frac{6\sqrt{3}}{6}$

$= \sqrt{3}$

Answer: $\mathbf{\sqrt{3}}$

5 Sets of numbers: lists and counting

ξ = integers from 1 to 24

A = the factors of 30

B = the multiples of 2

(a) List the elements of $A \cap B'$.

(b) Find $n(A' \cap B')$.

A = 1, 2, 3, 5, 6, 10, 15

B = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24

(a) $A \cap B'$ = 1, 3, 5, 15

(b) $A \cup B$ = 1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 15, 16, 18, 20, 22, 24 which has 16 elements, so $n(A' \cap B') = 24 - 16 = 8$

Answer: (a) 1, 3, 5, 15 (b) 8

6 Show that a line is a tangent

The line $y = 9x + k$ is a tangent to the curve $y = -2x^2 + x - 4$.
Find the value of the constant k.

$\frac{dy}{dx} = -4x + 1 = 9$, so $x = -2$

$y = -14$ when $x = -2$

The line passes through $(-2, -14)$: $-14 = 9 \times (-2) + k$, so $k = 4$

Answer: k = 4

7 Histograms

The histogram shows the masses of parcels delivered by a courier.
(a) Work out the total number of parcels.
(b) Estimate the number of parcels with a mass between 20 kg and 55 kg.

Frequency = frequency density × class width

10 to 30: $1.6 \times 20 = 32$

30 to 50: $1.6 \times 20 = 32$

50 to 60: $2 \times 10 = 20$

60 to 70: $1.2 \times 10 = 12$

(a) Total = 96

(b) 20 to 30: $1.6 \times 10 = 16$

Whole bar between: 32

50 to 55: $2 \times 5 = 10$, so estimate = 58

Answer: (a) 96 (b) 58

8 Increasing range for y = ax + b/x

$y = x + \frac{4}{x}$

Find the values of x for which y is increasing.

$\frac{dy}{dx} = 1 - \frac{4}{x^2}$

$\frac{dy}{dx} > 0$: $1 > \frac{4}{x^2}$

$x^2 > 4$ (x^2 is positive, so multiply through)

$x < -2$ or $x > 2$

Answer: $x < -2$ or $x > 2$

9 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 8 cm, top radius 2 cm and base radius 6 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+8} = \frac{2}{6}$

$6h = 2(h + 8)$, so $4h = 16$ and $h = 4$

Large cone: $\frac{1}{3} \times \pi \times 6^2 \times 12 = 144\pi$

Small cone: $\frac{1}{3} \times \pi \times 2^2 \times 4 = \frac{16}{3}\pi$

Frustum = $144\pi - \frac{16}{3}\pi = \frac{416}{3}\pi$ cm^3

Answer: $\frac{416}{3}\pi$ cm^3

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10 Inverse functions

$f(x) = \frac{x + 7}{3x - 2}$

(a) Find $f^{-1}(x)$.

(b) Find the value of $f^{-1}(8)$.

(a) Let $y = \frac{x + 7}{3x - 2}$. Then $y(3x - 2) = x + 7$

$3xy - 2y = x + 7$

Collect x terms: $3xy - x = 7 + 2y$

Factorise: $x(3y - 1) = 2y + 7$, so $x = \frac{2y + 7}{3y - 1}$

$f^{-1}(x) = \frac{2x + 7}{3x - 1}$

(b) $f^{-1}(8) = \frac{2 \times 8 + 7}{3 \times 8 - 1} = \frac{23}{23} = 1$

Answer: (a) $f^{-1}(x) = \frac{2x + 7}{3x - 1}$ (b) 1