Day 90 **IGCSE 10-a-day: Model Solutions**

1 Conditional probability from a two-way table

The table shows how 95 adults choose a drink.

A person who chose Tea is picked at random. Work out the probability that this person is in the group "Women".

Total who chose Tea = 19 + 29 = 48

Women who chose Tea = 29

Probability = $\frac{29}{48}$

Answer: $\mathbf{\frac{29}{48}}$

2 Weighted percentages

The ratio of men to women in a school is 1 : 2.

70% of the men and 40% of the women own a car.

What percentage of all of these students own a car?

Imagine 60 students in total: 20 men and 40 women

70% of 20 = 14 and 40% of 40 = 16

Together: 30 out of 60

30 ÷ 60 × 100 = 50%

Answer: 50%

3 Maximum speed from velocity

A particle moves in a straight line. Its velocity, v m/s, at time t seconds is given by

$v = 24t - 4t^2$ for $0 \leq t \leq 6$

Find the time at which the speed of the particle is greatest, and this greatest speed.

$a = \frac{dv}{dt} = -8t + 24$

Greatest speed when a = 0: $-8t + 24 = 0$, so t = 3

$\frac{d^2v}{dt^2} = -8 < 0$, so this is a maximum

$v = 24 \times 3 - 4 \times 3^2 = 36$

Answer: t = 3 s, greatest speed 36 m/s

4 Matrix of a transformation

Write down the matrix that represents each transformation.

(a) Stretch, scale factor 3, x-axis invariant

(b) Stretch, scale factor 3, y-axis invariant

(a) Find where (1, 0) and (0, 1) go: they are the columns of the matrix

$\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$

Answer: (a) $\mathbf{\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}}$ **(b)** $\mathbf{\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}}$

5 Stationary points of y = ax + b/x

$y = 3x + \frac{48}{x}$

Find the coordinates of the stationary points and determine whether each is a maximum or a minimum.

$\frac{dy}{dx} = 3 - \frac{48}{x^2}$

$\frac{dy}{dx} = 0$: $\frac{48}{x^2} = 3$, so $x^2 = 16$ and $x = 4$ or $x = -4$

$y = 24$ when $x = 4$ and $y = -24$ when $x = -4$

$\frac{d^2y}{dx^2} = \frac{96}{x^3}$

$x = 4$: $\frac{d^2y}{dx^2} = \frac{3}{2}$ so minimum; $x = -4$: $\frac{d^2y}{dx^2} = -\frac{3}{2}$ so maximum

Answer: (4, 24) minimum; (−4, −24) maximum

6 Trigonometric graphs

The diagram shows the graph of $y = a \sin x° + c$ for $0 \leq x \leq 360$.
Some points on the graph are labelled.
Find the value of a and the value of c.

At $x = 90$, $\sin x° = 1$, so $a + c = 8$
At $x = 270$, $\sin x° = -1$, so $-a + c = 2$
Add: $2c = 10$, so $c = 5$
Subtract: $2a = 6$, so $a = 3$
Answer: a = 3, c = 5

7 Pythagoras with algebra

ABC is a right-angled triangle with sides $(2x - 1)$ cm, $(x + 2)$ cm and $(x + 3)$ cm. The longest side is $(x + 3)$ cm.
(a) Find the value of x.
(b) Work out the area of the triangle.
The diagram is not drawn accurately.

(a) Pythagoras: $(2x - 1)^2 + (x + 2)^2 = (x + 3)^2$
Expand and collect terms: $4x^2 - 6x - 4 = 0$
Factorise: $2(x - 2)(2x + 1) = 0$
$x = 2$ or $x = -\frac{1}{2}$, and x must be positive, so $x = 2$

(b) The two shorter sides are 3 cm and 4 cm
Area $= \frac{1}{2} \times 3 \times 4 = 6$ cm^2

Answer: (a) x = 2 (b) 6 cm^2

8 Multiplying and dividing surds

Simplify fully $\frac{2\sqrt{75}}{\sqrt{3}}$

$\sqrt{75} = 5\sqrt{3}$
$\frac{2\sqrt{75}}{\sqrt{3}} = \frac{10\sqrt{3}}{\sqrt{3}}$
$= 10$
Answer: 10

9 Area of triangle from a tangent

The tangent to the curve $y = -3x^2 + 3x - 6$ at the point where $x = -1$ meets the axes at A and B.
Work out the area of triangle OAB, where O is the origin.

$y = -12$ and $\frac{dy}{dx} = -6x + 3$, so the gradient is 9

Tangent: $y = 9x - 3$

y-axis: $(0, -3)$. x-axis: $0 = 9x - 3$ so $x = \frac{1}{3}$

Area $= \frac{1}{2} \times 3 \times \frac{1}{3}$

$= \frac{1}{2}$

Answer: $\frac{1}{2}$ square units

10 Probability and quadratic equations

A bag contains some beads. 5 of them are blue and the rest are black.
Two beads are taken at random without replacement.

The probability that both are blue is $\frac{5}{14}$.

Work out how many beads are black.

Let there be n beads altogether

P(both blue) $= \frac{5}{n} \times \frac{4}{n-1} = \frac{20}{n(n-1)}$

$\frac{20}{n(n-1)} = \frac{5}{14}$, so $n(n-1) = 56$, giving $n^2 - n - 56 = 0$

$(n - 8)(n + 7) = 0$, so $n = 8$ (n cannot be negative)

Number of beads that are black $= 8 - 5 = 3$

Answer: 3