Day 9 **IGCSE 10-a-day: Model Solutions**

1 Arithmetic sequences

The 6th term of an arithmetic sequence is 44.
The 9th term of the sequence is 56.
(a) Find an expression for the nth term of the sequence.
(b) Is 114 a term of the sequence? You must show your working.

(a) Common difference = $(56 - 44) \div (9 - 6) = 4$
nth term = $4n + c$. For $n = 6$: $4 \times 6 + c = 44$, so $c = 20$
nth term = $4n + 20$

(b) Solve $4n + 20 = 114$: $4n = 94$, so $n = \frac{47}{2}$

n is not a whole number, so 114 is not a term
Answer: (a) $4n + 20$ (b) No

2 Local maximum and minimum values

$f(x) = 2x^3 - 6x + 7$
Find the y coordinate of the local maximum point and the y coordinate of the local minimum point.

$f'(x) = 6x^2 - 6 = 6(x - 1)(x + 1)$
$f'(x) = 0$ gives $x = -1$ or $x = 1$
$f''(x) = 12x$: $f''(-1) = -12 < 0$ (maximum) and $f''(1) = 12 > 0$ (minimum)
$f(-1) = 11$ and $f(1) = 3$
Answer: Local maximum y = 11, local minimum y = 3

3 Function values and composites

$f(x) = -2x - 8$ and $g(x) = 2x^2 - 3$.
(a) Work out $g(-1)$.
(b) Work out $fg(-4)$.

(a) $g(-1) = -1$
(b) $g(-4) = 29$, so $fg(-4) = f(29) = -66$
Answer: (a) -1 (b) -66

4 Error intervals

The distance to a village is 3.1 km, correct to 2 significant figures.
Write down the error interval for the distance to a village, using d.

The value has been rounded to the nearest 0.1
Half of this is 0.05, so go 0.05 below and 0.05 above
$3.1 - 0.05 = 3.05$ and $3.1 + 0.05 = 3.15$
$3.05 \leq d < 3.15$
Answer: $3.05 \leq d < 3.15$

5 Show that dy/dx = ...

$y = \frac{(x + 5)(3x - 4)}{x}$ $(x \neq 0)$

(a) Show that $\frac{dy}{dx} = \frac{3x^2 + 20}{x^2}$.

(b) Work out the gradient of the curve when $x = -1$.

Expand the brackets: $(x + 5)(3x - 4) = 3x^2 + 11x - 20$, then divide each term by x: $y = 3x + 11 - 20x^{-1}$

$\frac{dy}{dx} = 3 + \frac{20}{x^2}$

$= \frac{3x^2 + 20}{x^2}$

When $x = -1$: $\frac{dy}{dx} = 23$

Answer: (b) 23

6 Expanding three brackets

Expand and simplify $(x - 5)(x - 2)(x + 3)$

Expand the first two brackets: $(x - 5)(x - 2) = x^2 - 7x + 10$

Multiply by $(x + 3)$: $x(x^2 - 7x + 10) + 3(x^2 - 7x + 10)$

$= x^3 - 4x^2 - 11x + 30$

Answer: $x^3 - 4x^2 - 11x + 30$

7 Exact trigonometric ratios from a given ratio

x is an acute angle and $\cos x = \frac{3}{10}$.

(a) Find the exact value of sin x.

(b) Find the exact value of tan x. Give each answer as a fraction or a surd. Do not use a calculator.

(a) $\sin^2 x + \cos^2 x = 1$, so $\sin^2 x = 1 - (\frac{3}{10})^2 = \frac{91}{100}$

x is acute, so sin x is positive: $\sin x = \frac{\sqrt{91}}{10}$

(b) $\tan x = \frac{\sin x}{\cos x} = \frac{\sqrt{91}}{10} \div \frac{3}{10} = \frac{\sqrt{91}}{3}$

Answer: (a) $\frac{\sqrt{91}}{10}$ (b) $\frac{\sqrt{91}}{3}$

8 Bearings with sine and cosine rules

A ship sails 9 km from P to Q on a bearing of 068°.

It then sails 17 km from Q to R on a bearing of 192°.

(a) Work out PR (3 s.f.). (b) Work out the bearing of R from P (nearest degree). (Calculator)

Angle PQR = 068° + 180° − 192° = 56°

$PR^2 = 9^2 + 17^2 - 2 \times 9 \times 17 \times \cos 56°$ so PR = 14.1 km

$\cos QPR = \frac{9^2 + 14.103^2 - 17^2}{2 \times 9 \times 14.103}$, angle QPR = 92.06°

Bearing of R from P = 068° + 92.06° = 160°

Answer: (a) 14.1 km (b) 160°

9 Joint and inverse variation

F is directly proportional to the square of m and inversely proportional to d.

When m = 1 and d = 2, F = 12.

Work out the value of F when m = 2 and d = 2.

$F = \frac{km^2}{d}$

$12 = \frac{k \times 1}{2}$, so $k = 12 \times 2 \div 1 = 24$

$F = \frac{24 \times 4}{2} = 48$

Answer: F = 48

10 Number of solutions of f(x) = k

The diagram shows a sketch of the curve $y = -x^3 + 12x - 8$ and the line y = k.

The line meets the curve at three different points.

Find the range of possible values of k.

$\frac{dy}{dx} = -3x^2 + 12 = 0$ gives x = −2 and x = 2

y = −24 at x = −2 and y = 8 at x = 2

The line crosses three times when it lies between the two turning-point values

$-24 < k < 8$

Answer: $-24 < k < 8$