Day 89 **IGCSE 10-a-day: Model Solutions**

1 Matrix times vector and order

$B = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$ and $C = \begin{pmatrix} -3 & 1 \end{pmatrix}$.

(a) Work out CB.

(b) Write down the order of the matrix BC.

(a) $CB = \begin{pmatrix} (-3) \times (-5) + 1 \times 1 \end{pmatrix} = \begin{pmatrix} 16 \end{pmatrix}$

(b) B is 2 × 1 and C is 1 × 2, so BC is 2 × 2

Answer: (a) $\begin{pmatrix} 16 \end{pmatrix}$ (b) 2 × 2

2 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where

$s = t^3 - 8t^2 + 3t + 3$ $(t > 0)$

Find (a) the velocity of P, (b) the acceleration of P, when t = 4.

$v = \frac{ds}{dt} = 3t^2 - 16t + 3$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 6t - 16$

When t = 4: $v = -13$ and $a = 8$

Answer: (a) -13 m/s (b) 8 m/s^2

3 Rational and irrational numbers

Here are six numbers.

$2\pi - \pi$, $\sqrt{2} \times \sqrt{6}$, $(2 + \sqrt{5})(2 - \sqrt{5})$, $\sqrt{8} \times \sqrt{2}$, $\frac{\sqrt{20}}{\sqrt{5}}$, $\pi \div 2$

Write down the rational numbers.

Simplify each number:

$2\pi - \pi = \pi$; $\sqrt{2} \times \sqrt{6} = \sqrt{12} = 2\sqrt{3}$; $(2 + \sqrt{5})(2 - \sqrt{5}) = 4 - 5 = -1$

$\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$; $\frac{\sqrt{20}}{\sqrt{5}} = \sqrt{4} = 2$; $\pi \div 2$ is irrational

Answer: $(2 + \sqrt{5})(2 - \sqrt{5})$, $\sqrt{8} \times \sqrt{2}$, $\frac{\sqrt{20}}{\sqrt{5}}$

4 Turning point of a quadratic by differentiation

$f(x) = 3x^2 - 18x - 10$

Use differentiation to find the coordinates of the turning point of the graph of y = f(x). State whether it is a maximum or a minimum.

$f'(x) = 6x - 18$

$f'(x) = 0$: $6x - 18 = 0$, so $x = 3$

$f(3) = -37$

$f''(x) = 6$, which is positive, so it is a minimum

Answer: (3, -37), minimum

5 Perimeter and area of a running track

The diagram shows a running track made from a rectangle and two semicircles.

(a) Work out the total length of the boundary of the track.

(b) Work out the area enclosed by the track. Give your answers in terms of π.

The diagram is not drawn accurately.

(a) The two semicircles make one full circle of radius 15: circumference = $2 \times \pi \times 15 = 30\pi$

The two straight sides: $2 \times 100 = 200$

Boundary = $(200 + 30\pi)$ m

(b) The rectangle is 100 m by $2 \times 15 = 30$ m, so its area is $100 \times 30 = 3000$. Circle: $\pi \times 15^2 = 225\pi$

Area = $(3000 + 225\pi)$ m^2

Answer: (a) $(200 + 30\pi)$ m (b) $(3000 + 225\pi)$ m^2

6 Direct proportion formulae

C is directly proportional to d.
When d = 1, C = 12.
(a) Find a formula for C in terms of d.
(b) Work out the positive value of d when C = 60.

(a) C = kd
12 = k × 1, so k = 12 ÷ 1 = 12
C = 12d
(b) 60 = 12d, so d = 60 ÷ 12 = 5
Answer: (a) C = 12d (b) d = 5

7 Column vectors

The vectors $\begin{pmatrix} k \\ 12 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 3k \end{pmatrix}$ are parallel.
k is a positive number.
Work out the value of k.

Parallel vectors: one is a multiple of the other, so the ratio of the components is the same
$\frac{k}{4} = \frac{12}{3k}$
$k \times 3k = 12 \times 4$
$3k^2 = 48$
$k^2 = 16$
k = 4 (k is positive)
Answer: k = 4

8 Show that: trigonometric identities

Show that $(1 + \tan^2 A)\cos^2 A \equiv 1$.

tan = sin ÷ cos, so $(1 + \frac{\sin^2 A}{\cos^2 A})\cos^2 A$
$= \cos^2 A + \sin^2 A$
= 1 (since $\sin^2 + \cos^2 \equiv 1$)
Answer: Shown

9 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 + 9x + 14}{x^2 - 4x} \times \frac{x^2 - x}{x + 2}$

Factorise: $\frac{(x + 2)(x + 7)}{x(x - 4)} \times \frac{x(x - 1)}{(x + 2)}$
Cancel (x + 2) and x
$= \frac{(x - 1)(x + 7)}{(x - 4)}$
Answer: $\mathbf{\frac{(x - 1)(x + 7)}{(x - 4)}}$

10 Rate of cooling or growth

The temperature, T °C, of a metal bar t minutes after it is removed from a heat source is modelled by

$T = 20 + \frac{80}{t}$ $(t \geq 1)$

(a) Find $\frac{dT}{dt}$ when $t = 2$.

(b) Find the value of t when the temperature is falling at 5 °C per minute.

$T = 20 + 80t^{-1}$, so $\frac{dT}{dt} = -80t^{-2}$

When $t = 2$: $\frac{dT}{dt} = -20$

$-\frac{80}{t^2} = -5$, so $t^2 = 16$ and $t = 4$

Answer: (a) −20 °C/min (b) t = 4