Day 88 **IGCSE 10-a-day: Model Solutions**

1 Laws of indices

Write $\frac{3^{n-2} \times 9^n}{9^n}$ in the form 3^{an+c}

where a and c are integers.

$3 = 3^1$, $9 = 3^2$, $9 = 3^2$

$3^{n-2} = 3^{n-2}$, $9^n = 3^{2(n)}$, $9^n = 3^{2n}$

Exponent: $(n - 2) + 2(n) - 2n = n - 2$

Answer: a = 1, c = −2

2 Solving quadratics by factorising

Solve $4x^2 - 4x = 15$

Rearrange so that one side is 0: $4x^2 - 4x - 15 = 0$

Factorise: $(2x + 3)(2x - 5) = 0$

$2x + 3 = 0$ or $2x - 5 = 0$

$x = -\frac{3}{2}$ or $x = \frac{5}{2}$

Answer: $x = -\frac{3}{2}$ or $x = \frac{5}{2}$

3 Turning points of a cubic

A curve has equation $y = x^3 + 3x^2 - 9x + 4$.

Find the coordinates of the turning points of the curve. Use the second derivative to determine whether each is a maximum or a minimum.

$\frac{dy}{dx} = 3x^2 + 6x - 9 = 3(x + 3)(x - 1)$

$\frac{dy}{dx} = 0$ gives $x = -3$ or $x = 1$

$y = 31$ when $x = -3$ and $y = -1$ when $x = 1$

$\frac{d^2y}{dx^2} = 6x + 6$: at $x = -3$ it is -12 (maximum), at $x = 1$ it is 12 (minimum)

Answer: (−3, 31) maximum, (1, −1) minimum

4 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 60°.

(a) Work out the size of angle AOB.

(b) Work out the size of angle APB.

The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: $2 \times 60 = 120°$

(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)

Angle APB = $360 - 90 - 90 - 120 = 60°$

Answer: (a) 120° (b) 60°

5 Exponential model y = ab^t

$V = ab^n$, where a and b are positive constants.

When n = 2, V = 324. When n = 4, V = 262.44.

Find the values of a and b, and the value of V when n = 5.

Divide: $b^2 = 262.44 \div 324 = 0.81$

$b = 0.9$

$a \times 0.9^2 = 324$ so $a = 400$

$V = 400 \times 0.9^5 = 236.196$

Answer: a = 400, b = 0.9, V = 236.196

6 Particle at rest

A particle P moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where $s = 2t^3 - 18t^2 + 30t + 1$ $(t \geq 0)$

Find the distance between the two positions where P is at rest.

$v = \frac{ds}{dt} = 6t^2 - 36t + 30$

P is at rest when $v = 0$: $6t^2 - 36t + 30 = 0$

$6(t - 1)(t - 5) = 0$, so $t = 1$ or $t = 5$

$s(1) = 15$ and $s(5) = -49$

Distance $= |-49 - 15| = 64$

Answer: 64 m

7 Prime factors and indices

Write $9^2 \times 18^2$ as a product of powers of prime numbers.

$9 = 3^2$, $18 = 2 \times 3^2$

$9^2 = 3^4$

$18^2 = 2^2 \times 3^4$

Add the powers of each prime: $2^2 \times 3^8$

Answer: $2^2 \times 3^8$

8 Range and inverse of a quadratic

$g(x) = x^2 - 4x + 2$ for $x \leq 2$.

(a) Find the range of g.

(b) Find $g^{-1}(x)$.

(a) $g(x) = (x - 2)^2 - 2$

$x \leq 2$ includes the minimum point, so $g(x) \geq -2$

(b) $y = (x - 2)^2 - 2$, so $(x - 2)^2 = y + 2$

$x \leq 2$ means $x - 2$ is negative or zero, so $x = 2 - \sqrt{y + 2}$

$g^{-1}(x) = 2 - \sqrt{x + 2}$

Answer: (a) $g(x) \geq -2$ (b) $g^{-1}(x) = 2 - \sqrt{x + 2}$

9 Winning a match

Zane and Oscar play a match. The first to win 2 tennis sets wins the match.

The probability that Zane wins any one of the tennis sets is $\frac{4}{5}$.

Work out the probability that Zane wins the match.

Zane wins by WW (2 games) or WLW or LWW (3 games), where W means Zane wins that game

$P(WW) = \frac{4}{5} \times \frac{4}{5} = \frac{16}{25}$

$P(WLW) = P(LWW) = \frac{4}{5} \times \frac{1}{5} \times \frac{4}{5} = \frac{16}{125}$

$\text{Total} = \frac{16}{25} + 2 \times \frac{16}{125} = \frac{112}{125}$

Answer: $\frac{112}{125}$

10 Increasing for all x

$f(x) = x^3 + kx^2 + 12x + 6$, where k is a constant.

Given that f is an increasing function for all values of x, find the range of possible values of k.

$f'(x) = 3x^2 + 2kx + 12$

f is increasing for all x when $f'(x) > 0$ for all x, so the quadratic has no real roots

Discriminant: $(2k)^2 - 4 \times 3 \times 12 < 0$, so $4k^2 < 144$

$k^2 < 36$, so $-6 < k < 6$

Answer: $-6 < k < 6$