Day 87 **IGCSE 10-a-day: Model Solutions**

1 Angles in polygons

The interior angles of a pentagon are
$x°$, $(x + 20)°$, $(x + 15)°$, $(x + 30)°$, $(x + 25)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$5x + 90 = 540$, so $5x = 450$
$x = 90$
(b) The angles are 90°, 110°, 105°, 120°, 115°
The largest is 120°
Answer: (a) x = 90 (b) 120°

2 Fractional power functions

$f(x) = 3x - 2x^{-1/2} + 6x^{-3/2}$
Find $f'(x)$ and work out the value of $f'(1)$.

$3x \to 3$, $-2x^{-1/2} \to x^{-3/2}$, $6x^{-3/2} \to -9x^{-5/2}$
$f'(x) = 3 + x^{-3/2} - 9x^{-5/2}$
$f'(1)$: $\sqrt{1} = 1$, so $f'(1) = -5$
Answer: $f'(x) = 3 + x^{-3/2} - 9x^{-5/2}$, $f'(1) = -5$

3 Probabilities with an unknown

A biased four-sided dice lands on one of four outcomes with these probabilities.
1: 3x; 2: x; 3: 0.05; 4: 0.75
(a) Work out the value of x.
(b) The biased four-sided dice is used 100 times. Work out an estimate for the number of times it lands on 1.

(a) Probabilities add up to 1: $3x + x + 0.05 + 0.75 = 1$
$4x + 0.8 = 1$, so $4x = 0.2$
$x = 0.05$
(b) $P(1) = 3 \times 0.05 = 0.15$, so $100 \times 0.15 = 15$
Answer: (a) x = 0.05 (b) 15

4 Increasing or decreasing at a point

$y = 2x^3 + 9x^2 + 12x$
At the point where $x = -3$, is the curve increasing or decreasing? You must show your working.

$\frac{dy}{dx} = 6x^2 + 18x + 12$

When $x = -3$: $\frac{dy}{dx} = 12$

12 is positive, so the curve is increasing
Answer: increasing (gradient 12)

5 Factorising fully

Factorise fully $5x^3 - 45x$

Take out the common factor 5x: $5x(x^2 - 9)$
$x^2 - 9$ is a difference of two squares: $(x - 3)(x + 3)$
Answer: $5x(x - 3)(x + 3)$

6 Parallelogram and midpoint with vectors

ABCD is a parallelogram.
A = (−4, −1), B = (0, 5) and C = (0, 0).
Find the coordinates of D.

AD = **BC** because opposite sides are equal and parallel: $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$

D = A + **BC** = (−4, −1) + $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$

D = (−4, −6)

Answer: (−4, −6)

7 Line and curve intersections

Solve the simultaneous equations
$y = x^2 - x - 4$
$y = -2x + 16$

Substitute: $x^2 - x - 4 = -2x + 16$
Rearrange: $x^2 + x - 20 = 0$
Factorise: $(x + 5)(x - 4) = 0$, so $x = -5$ or $x = 4$
When $x = -5$: $y = -2 \times (-5) + 16 = 26$
When $x = 4$: $y = -2 \times 4 + 16 = 8$
Answer: x = −5, y = 26 and x = 4, y = 8

8 Normal in the form ax + by + c = 0

The point A lies on the curve $y = x^2 - 2x + 4$ where x = 0.
Find the equation of the normal at A. Give your answer in the form ax + by + c = 0 where a, b and c are integers.

y = 4 at x = 0 and gradient of curve = −2

Gradient of normal = $\frac{1}{2}$

$y - 4 = \frac{1}{2}x$

$y = \frac{1}{2}x + 4$

Multiply by 2: $x - 2y + 8 = 0$
Answer: x − 2y + 8 = 0

9 Combined transformations as matrices

P = $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ represents a rotation of 90° clockwise about the origin.

Q = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ represents a reflection in the line y = x.

(a) Find the matrix that represents P followed by Q.
(b) Describe the single transformation that this matrix represents.

(a) P followed by Q is the product QP (the first transformation is on the right)

QP = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(b) This matrix represents a reflection in the y-axis

Answer: (a) $\begin{pmatrix} \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$ **(b) Reflection in the y-axis**

10 Surds with unknowns

Given that $(a + \sqrt{13})^2 = 22 + 6\sqrt{13}$,
where a is a positive integer, find the value of a.

$(a + \sqrt{13})^2 = a^2 + 2a\sqrt{13} + 13$

Compare the $\sqrt{13}$ terms: $2a = 6$, so $a = 3$

Check the integer terms: $a^2 + 13 = 9 + 13 = 22$

Answer: a = 3