Day 85 **IGCSE 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 15:15 (London time).
It takes 8 hours 20 minutes.
When it is 12:00 in London it is 19:00 in Singapore.
At what time, in Singapore time, does the flight land? Say if it is on a different day.

Arrival in London time = 15:15 + 8 h 20 min = 23:35
Singapore is 7 hours ahead of London, so add 7 hours
06:35 the next day
Answer: 06:35 the next day

2 Divide then differentiate

$y = \frac{6x^4 + 7x^2 + 6x}{x}$

Find $\frac{dy}{dx}$.

Divide each term by x: $y = 6x^3 + 7x + 6$
$6x^3 \rightarrow 18x^2$, $7x \rightarrow 7$
$\frac{dy}{dx} = 18x^2 + 7$
Answer: $\mathbf{\frac{dy}{dx} = 18x^2 + 7}$

3 Two picks without replacement

A bag contains 4 green and 6 yellow beads.
Two beads are taken at random without replacement.
Work out the probability that both are green.

P(first green) = $\frac{4}{10}$

P(second green) = $\frac{3}{9}$

P(both green) = $\frac{4}{10} \times \frac{3}{9} = \frac{2}{15}$

Answer: $\mathbf{\frac{2}{15}}$

4 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 64°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle APB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 64 = 128°
(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angle APB = 360 − 90 − 90 − 128 = 52°
Answer: (a) 128° (b) 52°

5 Equations with indices

Solve $x^{-\frac{2}{3}} = \frac{1}{4}$

Write $\frac{1}{4}$ as a power of 2: $\frac{1}{4} = 2^{-2}$

$x^{-\frac{2}{3}} = 2^{-2}$, so $x = (2^{-2})^{-\frac{3}{2}}$
$x = 2^3 = 8$
Answer: x = 8

6 Variation: missing values

y is directly proportional to the square root of x.
When x = 9, y = 6.
When x = 36, y = a.
When x = b, y = 16.
Work out the values of a and b.

$y = k\sqrt{x}$, $k = 6 \div 3 = 2$
$a = 2 \times 6 = 12$
$16 = 2\sqrt{b}$ so $\sqrt{b} = 8$, $b = 64$
Answer: a = 12, b = 64

7 Show particle never at rest

A particle moves in a straight line. After t seconds its displacement from a fixed point is s metres, where
$s = 2t^3 - 18t^2 + 59t - 5$
(a) Show that the particle is never at rest.
(b) Find the least speed of the particle and the time it occurs.

$v = \frac{ds}{dt} = 6t^2 - 36t + 59 = 6(t - 3)^2 + 5$
A square is never negative, so $v \geq 5 > 0$ and P is never at rest
Least value of v occurs when t = 3, giving 5
Answer: (b) 5 m/s when t = 3 s

8 Rearranging when the subject appears twice

Make t the subject of $T(t + 3) = t - T$

Expand the bracket: $Tt + 3T = t - T$
Collect the t terms: $Tt - t = -3T - T$
Factorise: $t(T - 1) = -4T$
$t = \frac{4T}{1 - T}$
Answer: $t = \frac{4T}{1 - T}$

9 Ratio and expressions

3x = 5y
Work out the ratio $(4x - 5y) : (3x + 6y)$
Give your answer in its simplest form.

3x = 5y gives x : y = 5 : 3, so let x = 5t and y = 3t
$4(5t) - 5(3t) = 5t$
$3(5t) + 6(3t) = 33t$
5t : 33t = 5 : 33
Answer: 5 : 33

10 Average and instantaneous velocity

A particle moves in a straight line. Its displacement from O after t seconds is s metres, where
$s = -t^2 - 2t + 7$
(a) Work out the average velocity of the particle between t = 3 and t = 9.
(b) Find the time at which the velocity equals this average velocity.

$s(3) = -8$ and $s(9) = -92$
Average velocity $= \frac{-92 - (-8)}{9 - 3} = -14$
$v = \frac{ds}{dt} = -2t - 2$
$-2t - 2 = -14$ gives t = 6
Answer: (a) −14 m/s (b) t = 6 s