Day 84 **IGCSE 10-a-day: Model Solutions**

1 Estimating the mean from grouped data

The table shows the heights of 55 plants.
(a) Write down the modal class.
(b) Work out an estimate for the mean. Give your answer to 1 decimal place. (Calculator)

(a) Highest frequency is 19: class $10 < x \leq 20$
(b) Midpoints: 5, 15, 25, 35
$\Sigma fx = 9 \times 5 + 19 \times 15 + 17 \times 25 + 10 \times 35 = 1105$
Mean $\approx 1105 \div 55 = 20.1$
Answer: (a) $10 < x \leq 20$ (b) 20.1

2 Normal meets the x-axis

The normal to the curve $y = 3x^3 + x - 5$ at the point where $x = 0$ meets the x-axis at N.
Find the coordinates of N.

$y = -5$ and gradient of curve = 1
Gradient of normal $= -1$
$y + 5 = -x$
Put $y = 0$: $5 = -x$
$x = -5$
Answer: $(-5, 0)$

3 Median and quartiles from a list

Here are 15 numbers.
39, 15, 17, 32, 32, 21, 10, 2, 3, 30, 54, 57, 29, 52, 11
Find (a) the median (b) the interquartile range.

Ordered: 2, 3, 10, 11, 15, 17, 21, 29, 30, 32, 32, 39, 52, 54, 57
(a) Median is the 8th value = 29
(b) Lower quartile is the 4th value = 11; upper quartile is the 12th value = 39
IQR = 39 − 11 = 28
Answer: (a) 29 (b) 28

4 Fraction problems

A bag contains marbles.
$\frac{1}{6}$ of the marbles are yellow and $\frac{5}{8}$ of the marbles are black.
The rest of the marbles, 10 of them, are white.
How many black marbles are there?

Fraction that are white $= 1 - \frac{1}{6} - \frac{5}{8} = \frac{5}{24}$

$\frac{5}{24}$ of the total is 10

Total $= 10 \div \frac{5}{24} = 48$

Black marbles $= \frac{5}{8}$ of 48 = 30

Answer: 30

5 Local maximum and minimum values

$f(x) = 2x^3 - 24x - 3$
Find the y coordinate of the local maximum point and the y coordinate of the local minimum point.

$f'(x) = 6x^2 - 24 = 6(x - 2)(x + 2)$
$f'(x) = 0$ gives $x = -2$ or $x = 2$
$f''(x) = 12x$: $f''(-2) = -24 < 0$ (maximum) and $f''(2) = 24 > 0$ (minimum)
$f(-2) = 29$ and $f(2) = -35$
Answer: Local maximum y = 29, local minimum y = −35

6 Where a line meets the axes

The line L has equation $3x + 4y = -12$.
L crosses the x-axis at the point A and the y-axis at the point B.
(a) Find the coordinates of A and B.
(b) Work out the area of triangle OAB, where O is the origin.

(a) At A, $y = 0$: $3x = -12$, so $x = -4$. A is $(-4, 0)$
At B, $x = 0$: $4y = -12$, so $y = -3$. B is $(0, -3)$
(b) OA = 4 and OB = 3, and angle AOB = 90°

Area $= \frac{1}{2} \times 4 \times 3 = 6$

Answer: (a) A$(-4, 0)$, B$(0, -3)$ (b) 6

7 Column vectors

$\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

Work out the exact length of the vector $2\mathbf{a} + \mathbf{b}$. Give your answer as a simplified surd.

$2\mathbf{a} + \mathbf{b} = 2\begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 10 \\ 3 \end{pmatrix}$

Length $= \sqrt{(10)^2 + (3)^2} = \sqrt{109}$
$= \sqrt{109}$

Answer: $\sqrt{109}$

8 Inverse proportion

V is inversely proportional to r.
When r = 3, V = 4.
Find the value of V when r = 2.

$V = \frac{k}{r}$

$4 = \frac{k}{3}$, so $k = 4 \times 3 = 12$

$V = \frac{12}{2} = 6$

Answer: V = 6

9 Greatest and least values on an interval

$f(x) = x^3 - 6x^2 - 2$ for $-1 \leq x \leq 5$.
Find the greatest value and the least value of f(x) on this interval.

$f'(x) = 3x^2 - 12x = 0$ gives $x = 0$ or $x = 4$ (both in the interval)
$f(-1) = -9$, $f(0) = -2$, $f(4) = -34$, $f(5) = -27$
Compare the turning points and both end values
Answer: Greatest -2, least -34

10 Equations with algebraic fractions

Solve $\frac{2}{x+1} + \frac{5}{x+5} = \frac{2}{3}$

Multiply every term by $(x + 1)(x + 5)$: $2(x + 5) + 5(x + 1) = \frac{2}{3} \times (x + 1)(x + 5)$
Multiply by 3 to clear the fraction: $6(x + 5) + 15(x + 1) = 2(x + 1)(x + 5)$
Expand and rearrange so that one side is 0: $2x^2 - 9x - 35 = 0$
Factorise: $(x - 7)(2x + 5) = 0$
$x = -\frac{5}{2}$ or $x = 7$

Answer: $x = -\frac{5}{2}$ or $x = 7$