Day 83 **IGCSE 10-a-day: Model Solutions**

1 Angles in polygons

The interior angles of a pentagon are
$x°$, $(2x + 10)°$, $(x + 5)°$, $(2x - 20)°$, $(2x + 25)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$8x + 20 = 540$, so $8x = 520$
$x = 65$
(b) The angles are 65°, 140°, 70°, 110°, 155°
The largest is 155°
Answer: (a) x = 65 (b) 155°

2 Points with a given gradient on y = ax + b/x

A curve has equation $y = x + \frac{18}{x}$.

Find the coordinates of the points on the curve where the gradient is -1.

$\frac{dy}{dx} = 1 - \frac{18}{x^2}$

Set $\frac{dy}{dx} = -1$: $2 = \frac{18}{x^2}$

$x^2 = 9$, so $x = 3$ or $x = -3$
$y = 9$ when $x = 3$ and $y = -9$ when $x = -3$
Answer: (3, 9) and (−3, −9)

3 Solving quadratics by factorising

Solve $3x^2 + 10x - 25 = 0$

Factorise: $(x + 5)(3x - 5) = 0$
$x + 5 = 0$ or $3x - 5 = 0$
$x = -5$ or $x = \frac{5}{3}$

Answer: $x = -5$ or $x = \frac{5}{3}$

4 Rational and irrational numbers

Here are six numbers.

$\sqrt[3]{27}$, $2\pi - \pi$, $(2 + \sqrt{5})(2 - \sqrt{5})$, $\frac{\sqrt{20}}{\sqrt{5}}$, $\sqrt{3} \times \sqrt{5}$, $\pi \div 2$

Write down the irrational numbers.

Simplify each number:
$\sqrt[3]{27} = 3$; $2\pi - \pi = \pi$; $(2 + \sqrt{5})(2 - \sqrt{5}) = 4 - 5 = -1$
$\frac{\sqrt{20}}{\sqrt{5}} = \sqrt{4} = 2$; $\sqrt{3} \times \sqrt{5} = \sqrt{15}$; $\pi \div 2$ is irrational
Answer: $2\pi - \pi$, $\sqrt{3} \times \sqrt{5}$, $\pi \div 2$

5 Solving trigonometric equations

Solve $2 \sin x = \sqrt{3}$ for $0° \leq x \leq 360°$.
Do not use a calculator.

$\sin x = \frac{\sqrt{3}}{2}$

The related acute angle satisfies $\sin \theta = \frac{\sqrt{3}}{2}$, so $\theta = 60°$

sine is positive in the 1st and 2nd quadrants, so $x = 60°$ and $x = 180° - 60°$
$x = 60°$ and $x = 120°$
Answer: x = 60° or x = 120°

6 Cumulative frequency graphs

The cumulative frequency graph gives information about the times of 120 students to complete a puzzle.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 120 had a time greater than 75.

(a) Median at cumulative frequency 60, so median = 50
Lower quartile at 30, so LQ = 40. Upper quartile at 90, so UQ = 60
Interquartile range = 60 − 40 = 20
(b) At 75, cumulative frequency = 114
Number greater = 120 − 114 = 6
Answer: (a) median 50, IQR 20 (b) 6

7 Show that: consecutive terms of a quadratic sequence

The nth term of a sequence is $3n^2 + 7n + 5$.
(a) Show that the difference between the (n + 1)th term and the nth term is 6n + 10.
(b) Two consecutive terms of the sequence differ by 40. Find the value of n for the first of these two terms.

(a) (n + 1)th term = $3(n + 1)^2 + 7(n + 1) + 5 = 3n^2 + 13n + 15$
Subtract the nth term: $3n^2 + 13n + 15 - (3n^2 + 7n + 5) = 6n + 10$
(b) 6n + 10 = 40 so 6n = 30
n = 5
Answer: (b) n = 5

8 Minimum gradient of a curve

A curve has equation $y = 2x^3 + 18x^2 + 52x - 1$.
Find the minimum value of the gradient of the curve, and the value of x at which it occurs.

$\frac{dy}{dx} = 6x^2 + 36x + 52$

The gradient is least when $\frac{d^2y}{dx^2} = 0$: 12x + 36 = 0, so x = −3

$\frac{d^2y}{dx^2}$ increases through zero here (its derivative is 12 > 0), so this is a minimum

$\frac{dy}{dx} = -2$ when x = −3
Answer: minimum gradient −2 when x = −3

9 Range and inverse of a quadratic

$g(x) = x^2 - 4x + 5$ for $x \leq 2$.
(a) Find the range of g.
(b) Find $g^{-1}(x)$.

(a) $g(x) = (x - 2)^2 + 1$
$x \leq 2$ includes the minimum point, so $g(x) \geq 1$
(b) $y = (x - 2)^2 + 1$, so $(x - 2)^2 = y - 1$
$x \leq 2$ means $x - 2$ is negative or zero, so $x = 2 - \sqrt{y - 1}$
$g^{-1}(x) = 2 - \sqrt{x - 1}$
Answer: (a) $g(x) \geq 1$ (b) $g^{-1}(x) = 2 - \sqrt{x - 1}$

10 Rectangle under a parabola

The diagram shows the curve $y = 27 - x^2$. Rectangle ABCD has A, B on the x-axis and C, D on the curve. B is (x, 0).

(a) Show that the area of ABCD is $-2x^3 + 54x$.

(b) Find the maximum area of ABCD, showing that it is a maximum.

AB = 2x and BC = $27 - x^2$, so area = $2x(27 - x^2) = -2x^3 + 54x$

$\frac{dA}{dx} = -6x^2 + 54 = 0$, so $x^2 = 9$ and $x = 3$ ($x > 0$)

$\frac{d^2A}{dx^2} = -12x < 0$ when $x = 3$, so a maximum

Area = $54 \times 3 - 2 \times 3^3 = 108$

Answer: 108