Day 82 **IGCSE 10-a-day: Model Solutions**

1 Negative powers

$y = 6x^4 - \frac{2}{x}$

Find $\frac{dy}{dx}$.

Write negative powers as indices: $y = 6x^4 - 2x^{-1}$

$6x^4 \rightarrow 24x^3$, $-2x^{-1} \rightarrow 2x^{-2}$

$\frac{dy}{dx} = 24x^3 + 2x^{-2}$

Answer: $\mathbf{\frac{dy}{dx} = 24x^3 + \frac{2}{x^2}}$

2 Fractions

Work out $3\frac{1}{2} \times 2\frac{5}{6} - 1\frac{1}{2}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions

$\frac{7}{2} \times \frac{17}{6} = \frac{119}{12}$

$\frac{119}{12} - \frac{3}{2} = \frac{101}{12}$

$= 8\frac{5}{12}$

Answer: $\mathbf{8\frac{5}{12}}$

3 Perpendicular lines

The diagram shows the straight line L, which passes through the points A and B.
The line M is perpendicular to L and passes through B.
Find the equation of M in the form y = mx + c.

Gradient of L = $\frac{-2 - 0}{0 - 3} = \frac{2}{3}$

Perpendicular gradient = $-1 \div \frac{2}{3} = -\frac{3}{2}$

M passes through B(3, 0): $0 = (-\frac{3}{2}) \times 3 + c$, so $c = \frac{9}{2}$

$y = -\frac{3}{2}x + \frac{9}{2}$

Answer: $\mathbf{y = -\frac{3}{2}x + \frac{9}{2}}$

4 Rate of cooling or growth

The temperature, T °C, of a metal bar t minutes after it is removed from a heat source is modelled by

$T = 20 + \frac{32}{t}$ $(t \geq 1)$

(a) Find $\frac{dT}{dt}$ when t = 2.

(b) Find the value of t when the temperature is falling at 32 °C per minute.

$T = 20 + 32t^{-1}$, so $\frac{dT}{dt} = -32t^{-2}$

When t = 2: $\frac{dT}{dt} = -8$

$-\frac{32}{t^2} = -32$, so $t^2 = 1$ and t = 1

Answer: (a) −8 °C/min (b) t = 1

5 Cumulative frequency graph: median and quartiles

The cumulative frequency graph shows the times, in minutes, taken by 160 runners to finish a race.
(a) Find the median. (b) Find the interquartile range.
(c) How many runners took more than 70?

(a) Median at cumulative frequency 80: reading across gives 50
(b) Lower quartile at 40 gives 30, upper quartile at 120 gives 70
IQR = 70 − 30 = 40
(c) At 70 the cumulative frequency is 120, so 160 − 120 = 40
Answer: (a) 50 (b) 40 (c) 40

6 Speed-time graphs

The graph shows the speed of a car during 16 seconds.
(a) Work out the acceleration during the first 4 seconds.
(b) Work out the average speed over the whole 16 seconds.

(a) Acceleration = change in speed ÷ time = (24 − 8) ÷ 4 = 4 m/s^2
(b) Distance = area under the graph
Trapezium: $\frac{8+24}{2} \times 4 = 64$; rectangle: $24 \times 6 = 144$; triangle: $\frac{1}{2} \times 6 \times 24 = 72$
Total = 280 m, so average speed = 280 ÷ 16 = 17.5 m/s
Answer: (a) 4 m/s^2 (b) 17.5 m/s

7 Bounds in calculations

A rectangle has length 9.8 cm and width 11.1 cm, each correct to 1 decimal place.
(a) Work out the upper bound of the area.
(b) Work out the lower bound of the perimeter.

Bounds: 9.75 ≤ length < 9.85, 11.05 ≤ width < 11.15
(a) UB area = 9.85 × 11.15 = 109.8275
(b) LB perimeter = 2 × (9.75 + 11.05) = 41.6
Answer: (a) 109.8275 cm^2 (b) 41.6 cm

8 Loci and regions

ABCD is a rectangle with AB = 5 cm and AD = 9 cm.
A point P inside the rectangle is such that AP ≤ 4 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 4 cm: P is inside the quarter circle of radius 4 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves
Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 4^2 = 2\pi$ cm^2
Answer: 2π cm^2

9 Bounds and accuracy

a = 55.9 and b = 20.7, both correct to 3 significant figures.
Work out a × b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

55.85 ≤ a < 55.95; 20.65 ≤ b < 20.75
Lower bound = 55.85 × 20.65 = 1153.3025
Upper bound = 55.95 × 20.75 = 1160.9625
Both bounds round to 1200 to 2 significant figures, but not to 3 significant figures
Answer: 1200 (2 s.f.)

10 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where $s = -t^3 - 2t^2 - 2t$ $(t > 0)$

Find (a) the velocity of P, (b) the acceleration of P, when t = 1.

$$v = \frac{ds}{dt} = -3t^2 - 4t - 2$$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = -6t - 4$$

When t = 1: $v = -9$ and $a = -10$

Answer: (a) -9 m/s (b) -10 m/s^2