Day 81 **IGCSE 10-a-day: Model Solutions**

1 Direct variation

E is directly proportional to the square root of v.
When v = 25, E = 35.
(a) Find a formula for E in terms of v.
(b) Find the value of v when E = 49.

$E = k\sqrt{v}$
$35 = k \times \sqrt{25}$ so $k = 7$
$E = 7\sqrt{v}$
$49 = 7\sqrt{v}$ so $\sqrt{v} = 7$
$v = 49$
Answer: (a) $E = 7\sqrt{v}$ (b) $v = 49$

2 Mean of a changing set

The mean of 9 numbers is 61.
One number is removed and the mean of the remaining 8 numbers is 65.
Work out the number that was removed.

Total of the 9 numbers = $9 \times 61 = 549$
Total of the 8 remaining = $8 \times 65 = 520$
Number removed = $549 - 520 = 29$
Answer: 29

3 Similar triangles with algebra

In the diagram, DE is parallel to BC.
AD = x cm, DB = (x + 6) cm, DE = 6 cm and BC = 14 cm.
(a) Find the value of x.
(b) Work out the length of AB.
The diagram is not drawn accurately.

(a) Triangles ADE and ABC are similar, so $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{x}{2x + 6} = \frac{6}{14}$
$14x = 6(2x + 6)$
$14x = 12x + 36$
$2x = 36$
$x = 18$
(b) AB = $2x + 6 = 42$ cm
Answer: (a) $x = 18$ (b) 42 cm

4 Rate of change of area or volume

A cylinder has radius r cm and fixed height 5 cm. Its volume is $V = 5\pi r^2$ cm^3.
(a) Find $\frac{dV}{dr}$ when r = 7.
(b) Find the value of r when $\frac{dV}{dr} = 80\pi$.

$\frac{dV}{dr} = 10\pi r$
When r = 7: $\frac{dV}{dr} = 70\pi$
$10\pi r = 80\pi$, so $10r = 80$ and $r = 8$ (lengths are positive)
Answer: (a) 70π (b) $r = 8$

5 Relative speed

Jamal sets off at 9:00 and cycles at 14 km/h along a path.
Hassan sets off from the same place at 9:45 and cycles at 32 km/h along the same path.
At what time does Hassan catch up with Jamal?

When Hassan starts, Jamal is $0.75 \times 14 = 10.5$ km ahead
Hassan gains $32 - 14 = 18$ km every hour

Time to catch up = $10.5 \div 18 = \frac{7}{12}$ hours = 35 minutes

9:45 + 35 minutes = 10:20

Answer: 10:20

6 Stationary point of a quartic

$y = x^4 + 32x$
Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 4x^3 + 32$

$\frac{dy}{dx} = 0$: $4x^3 + 32 = 0$, so $x^3 = -8$

$x = -2$ and $y = -48$

$\frac{d^2y}{dx^2} = 12x^2$

At $x = -2$: $\frac{d^2y}{dx^2} = 48$, so a minimum

Answer: $(-2, -48)$, minimum

7 Quadratic formula and surds

Solve $2x^2 + 7x - 2 = 0$
Give your answers in surd form.

Use the quadratic formula with $a = 2$, $b = 7$, $c = -2$

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \times 2 \times (-2)}}{2 \times 2}$$

$$x = \frac{-7 \pm \sqrt{65}}{4}$$

Answer: $x = \frac{-7 \pm \sqrt{65}}{4}$

8 Frequency density from a table

The table shows the masses of 129 parcels.
(a) Work out the frequency density for the class $5 < x \leq 15$.
(b) Estimate how many were greater than 49 kg.

(a) Frequency density = frequency ÷ class width = $11 \div 10 = 1.1$
(b) In $45 < x \leq 50$, the part above 49 is 1 out of 5
Estimate = $4 \times 1 = 4$
Add the higher classes: no higher classes, total = 4
Answer: (a) 1.1 (b) 4

9 Greatest speed from displacement

A particle moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where $s = -2t^3 + 30t^2 + 10t$ for $t \geq 0$

Find the greatest speed of the particle, and the value of t at which it occurs.

$v = \frac{ds}{dt} = -6t^2 + 60t + 10$

$a = \frac{dv}{dt} = -12t + 60$

Maximum v when a = 0: $-12t + 60 = 0$, so $t = 5$

$\frac{d^2v}{dt^2} = -12 < 0$, so v is a maximum

$v = 160$

Answer: 160 m/s when t = 5 s

10 Surds

Given that $6\sqrt{5} = \sqrt{n}$,

find the value of n.

$6\sqrt{5} = \sqrt{36} \times \sqrt{5} = \sqrt{180}$

$n = 180$

Answer: n = 180