Day 80 **IGCSE 10-a-day: Model Solutions**

1 Simplifying surds

Write $\sqrt{125} - \sqrt{5} + \sqrt{45}$ in the form $a\sqrt{5}$ where a is an integer.

$\sqrt{125} = 5\sqrt{5}$; $\sqrt{45} = 3\sqrt{5}$

$(5 - 1 + 3)\sqrt{5} = 7\sqrt{5}$

Answer: $7\sqrt{5}$

2 Negative powers

$y = -3x^4 - 5x - \frac{1}{x}$

Find $\frac{dy}{dx}$.

Write negative powers as indices: $y = -3x^4 - 5x - x^{-1}$

$-3x^4 \rightarrow -12x^3$, $-5x \rightarrow -5$, $-x^{-1} \rightarrow x^{-2}$

$\frac{dy}{dx} = -12x^3 - 5 + x^{-2}$

Answer: $\frac{dy}{dx} = -12x^3 - 5 + \frac{1}{x^2}$

3 Arc length and sector area

A sector of a circle has radius 15 cm.
The length of the arc of the sector is 5π cm.
(a) Work out the angle of the sector.
(b) Work out the area of the sector. Give your answer to (b) in terms of π.

(a) Arc length = $\frac{\theta}{360} \times 2\pi \times 15$, so $\frac{\theta}{360} \times 30 = 5$

$\theta = 5 \times \frac{360}{30} = 60°$

(b) Area = $\frac{60}{360} \times \pi \times 15^2 = \frac{75}{2}\pi$ cm^2

Answer: (a) 60° (b) $\frac{75}{2}\pi$ cm^2

4 Second derivative and nature at a given point

$y = 2x^3 + 6x^2 - 18x$

Use the second derivative to show that the curve has a minimum at x = 1.

$\frac{dy}{dx} = 6x^2 + 12x - 18$

When x = 1: $\frac{dy}{dx} = 0$, so it is a stationary point

$\frac{d^2y}{dx^2} = 12x + 12$

When x = 1: $\frac{d^2y}{dx^2} = 24$, which is positive

So there is a minimum at x = 1

Answer: minimum at x = 1

5 Variation: missing values

T is inversely proportional to the square of m.
When m = 4, T = 2.25.
When m = 6, T = a.
When m = b, T = 0.5625.
Work out the values of a and b.

$T = \frac{k}{m^2}$, $k = 2.25 \times 16 = 36$

$a = \frac{36}{36} = 1$

$0.5625 = \frac{36}{b^2}$ so $b^2 = 64$, $b = 8$

Answer: a = 1, b = 8

6 Gradient where the curve crosses the x-axis

A curve has equation $y = (x - 2)(x + 2)(x + 3)$.
Work out the gradient of the curve at each point where it crosses the x-axis.

Expand: $y = x^3 + 3x^2 - 4x - 12$

$\frac{dy}{dx} = 3x^2 + 6x - 4$

The curve crosses the x-axis at $x = -3$, -2 and 2
Gradients: $x = -3$ gives 5, $x = -2$ gives -4, $x = 2$ gives 20
Answer: 5 at $x = -3$, -4 at $x = -2$, 20 at $x = 2$

7 HCF and LCM

$A = 2^3 \times 3^2 \times 11$

$B = 2^2 \times 11^2$

Write down, as a product of prime factors,
(a) the HCF of A and B
(b) the LCM of A and B

HCF: take the lowest power of each prime that appears in both numbers
$= 2^2 \times 11$
LCM: take the highest power of every prime that appears
$= 2^3 \times 3^2 \times 11^2$

Answer: (a) $2^2 \times 11$ (b) $2^3 \times 3^2 \times 11^2$

8 Transformed curve from a diagram

The diagram shows the graph of y = f(x) with a minimum point at (0, 1).
The dashed curve C is the image of y = f(x) after a translation.
The minimum point of C is (−2, 0).
Write down the equation of C in terms of f.

The minimum point has moved from (0, 1) to (−2, 0)

Translation by $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$

x moves left by 2: f(x + 2). y moves down by 1: − 1
Answer: y = f(x + 2) − 1

9 Changing the subject

Make l the subject of $T = 5l^2 - 5$
where l is positive.

Add 5: $T + 5 = 5l^2$

Divide by 5: $l^2 = \frac{T + 5}{5}$

Square root (positive): $l = \sqrt{\frac{T + 5}{5}}$

Answer: $l = \sqrt{\frac{T + 5}{5}}$

10 Successive percentage change

The value of a car is increased by 40%.
The new amount is then decreased by 40%.
Work out the overall percentage change.

Multipliers: 1.4 and 0.6
1.4 × 0.6 = 0.84
0.84 is a decrease of 16%
Answer: 16% decrease