Day 8 **IGCSE 10-a-day: Model Solutions**

1 Substitution

Work out the value of $3x - y^3$
when $x = -5$, $y = -3$ and $z = 4$.

Substitute the values: $3 \times (-5) - (-3)^3$
$= -15 - (-27)$
$= 12$
Answer: 12

2 Solving trigonometric equations

Solve $7 \tan x + 3 = 0$ for $0° \leq x \leq 360°$. (Calculator)
Give your answers correct to 1 decimal place.

$\tan x = -\frac{3}{7}$

Related acute angle: $\tan^{-1}(\frac{3}{7}) = 23.2°$

tangent is negative in the 2nd and 4th quadrants, so $x = 180° - 23.2°$ and $x = 360° - 23.2°$
$x = 156.8°$ and $x = 336.8°$
Answer: $x = 156.8°$ or $x = 336.8°$

3 Matrix times vector and order

$B = \begin{pmatrix} -4 \\ 4 \end{pmatrix}$ and $C = \begin{pmatrix} -2 & 5 \end{pmatrix}$.

(a) Work out CB.
(b) Write down the order of the matrix BC.

(a) $CB = \begin{pmatrix} (-2) \times (-4) + 5 \times 4 \end{pmatrix} = \begin{pmatrix} 28 \end{pmatrix}$
(b) B is 2×1 and C is 1×2, so BC is 2×2
Answer: (a) $\begin{pmatrix} 28 \end{pmatrix}$ (b) 2×2

4 Greatest speed from displacement

A particle moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where
$s = -2t^3 + 24t^2 + 5t$ for $t \geq 0$
Find the greatest speed of the particle, and the value of t at which it occurs.

$v = \frac{ds}{dt} = -6t^2 + 48t + 5$

$a = \frac{dv}{dt} = -12t + 48$

Maximum v when $a = 0$: $-12t + 48 = 0$, so $t = 4$

$\frac{d^2v}{dt^2} = -12 < 0$, so v is a maximum

$v = 101$
Answer: 101 m/s when t = 4 s

5 Quadratic formula and surds

Solve $3x^2 + 9x + 5 = 0$
Give your answers in surd form.

Use the quadratic formula with $a = 3$, $b = 9$, $c = 5$

$x = \frac{-9 \pm \sqrt{9^2 - 4 \times 3 \times 5}}{2 \times 3}$

$x = \frac{-9 \pm \sqrt{21}}{6}$

Answer: $x = \frac{-9 \pm \sqrt{21}}{6}$

6 Normal meets the x-axis

The normal to the curve $y = 2x^2 - 3x - 5$ at the point where $x = 1$ meets the x-axis at N.
Find the coordinates of N.

$y = -6$ and gradient of curve = 1
Gradient of normal $= -1$
$y + 6 = -(x - 1)$
Put $y = 0$: $6 = -(x - 1)$
$x = -5$
Answer: $(-5, 0)$

7 Inverse proportion

V is inversely proportional to r.
When r = 3, V = 3.
Find the value of V when r = 1.

$V = \frac{k}{r}$

$3 = \frac{k}{3}$, so $k = 3 \times 3 = 9$

$V = \frac{9}{1} = 9$

Answer: V = 9

8 Rationalising the denominator

Write $\frac{3 + \sqrt{6}}{3 - \sqrt{6}}$ in the form $p + q\sqrt{6}$
where p and q are rational numbers.

Multiply top and bottom by $(3 + \sqrt{6})$:
Numerator: $(3 + \sqrt{6})(3 + \sqrt{6}) = 15 + 6\sqrt{6}$
Denominator: $9 - 6 = 3$
$= 5 + 2\sqrt{6}$
Answer: $5 + 2\sqrt{6}$

9 Area of triangle from a tangent

The tangent to the curve $y = -x^2 + x - 4$ at the point where $x = 0$ meets the axes at A and B.
Work out the area of triangle OAB, where O is the origin.

$y = -4$ and $\frac{dy}{dx} = -2x + 1$, so the gradient is 1
Tangent: $y = x - 4$
y-axis: $(0, -4)$. x-axis: $0 = x - 4$ so $x = 4$
Area $= \frac{1}{2} \times 4 \times 4$
$= 8$
Answer: 8 square units

10 Bounds: finding an unknown length

A rectangle has area 60 cm^2, correct to 2 significant figures.
Its length is 8.400000000000000355271367880050092935562133789062 5 cm, correct to 2 significant figures.
Work out the upper bound of the width of the rectangle. Give your answer correct to 3 significant figures. (Calculator)

Width = area ÷ length
Area: $59.5 \leq A < 60.5$. Length: $8.350000000000000355271367880050092935562133789062 5 \leq l < 8.450000000000000355271367880050092935562133789062 5$
UB width = UB area ÷ LB length = 60.5 ÷ 8.350000000000000355271367880050092935562133789062 5 = 7.25
Answer: 7.25 cm