Day 79 **IGCSE 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 07:35 (London time).
It takes 13 hours 30 minutes.
When it is 12:00 in London it is 19:00 in Singapore.
At what time, in Singapore time, does the flight land? Say if it is on a different day.

Arrival in London time = 07:35 + 13 h 30 min = 21:05
Singapore is 7 hours ahead of London, so add 7 hours
04:05 the next day
Answer: 04:05 the next day

2 Capture and recapture

30 fish in a lake are caught, tagged and put back.
Later, a second sample of 30 fish is caught. 3 of these are tagged.
(a) Work out an estimate for the number of fish in a lake.
(b) State one assumption you have made.

(a) The proportion tagged in the second sample is 3 out of 30

This should match the proportion tagged in the whole population: $\frac{3}{30} = \frac{30}{N}$

N = 30 × 30 ÷ 3 = 300
(b) No fish are born, die or leave between the samples, and the fish mix evenly
Answer: (a) 300 (b) The population stays the same between samples

3 Parallel vectors

A = (−4, 4), B = (−1, 5), C = (0, 4) and D = (k, 7).
AB is parallel to **CD**.
Find the value of k.

$\mathbf{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\mathbf{CD} = \begin{pmatrix} k \\ 3 \end{pmatrix}$
Vertical parts: 3 = λ × 1, so λ = 3
Horizontal: k = 3 × 3 = 9
k = 9
Answer: k = 9

4 Increasing and decreasing intervals

$y = -2x^3 + 18x^2 - 48x - 7$
Find the range of values of x for which y is decreasing.

$\frac{dy}{dx} = -6x^2 + 36x - 48 = -6(x - 2)(x - 4)$

$\frac{dy}{dx} = 0$ when x = 2 and x = 4

The graph of $\frac{dy}{dx}$ against x is a ∩-shaped quadratic

y is increasing when $\frac{dy}{dx} > 0$ and decreasing when $\frac{dy}{dx} < 0$

So y is decreasing when x < 2 or x > 4
Answer: x < 2 or x > 4

5 Finding a percentage rate

A machine was bought for £14,500.
Each year its value decreases by the same percentage.
After 3 years it is worth £7,424.
Work out the percentage decrease each year.

Multiplier for 3 years = £7,424 ÷ £14,500 = 0.512
Yearly multiplier = the cube root of 0.512 = 0.8
20% decrease each year
Answer: 20%

6 Tangents parallel to a line

A curve has equation $y = x^3 + 3x^2 - 4x - 8$.
Find the equations of the two tangents to the curve that are parallel to the line $y = -4x - 8$.

Parallel lines have equal gradients: gradient $= -4$

$\frac{dy}{dx} = 3x^2 + 6x - 4 = -4$, so $3x^2 + 6x = 0$

$3x(x + 2) = 0$ gives $x = -2$ or $x = 0$

At $x = -2$: $y = 4$, tangent $y - 4 = -4(x - (-2))$

At $x = 0$: $y = -8$, tangent $y - (-8) = -4(x - 0)$

Answer: $y = -4x - 4$ and $y = -4x - 8$

7 Tangent to a circle

A circle has equation $(x + 2)^2 + (y - 4)^2 = 50$.
The point $P(-3, -3)$ lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form $ax + by = c$, where a, b and c are integers.

The centre is $(-2, 4)$. The gradient of the radius to P is $\frac{-3 - 4}{-3 - (-2)} = 7$

The tangent is perpendicular to the radius, so its gradient is $-\frac{1}{7}$. Using P: $y + 3 = -\frac{1}{7}(x + 3)$

Multiply by 7: $7(y + 3) = -1(x + 3)$

Expand and rearrange: $x + 7y = -24$

Answer: $x + 7y = -24$

8 Shaded areas: exact answers

The diagram shows a square inside a circle of radius 2 cm.
The four vertices of the square lie on the circle.
Work out the area of the shaded region. Give your answer in terms of π.

The diagonal of the square is a diameter of the circle: 4 cm

Area of the square $= \frac{1}{2} \times \text{diagonal} \times \text{diagonal} = \frac{1}{2} \times 4 \times 4 = 8$

Area of circle $= \pi \times 2^2 = 4\pi$

Shaded area $= 4\pi - 8$ cm^2

Answer: $(4\pi - 8)$ cm^2

9 Normal to y = kx^2: show that

P is the point $(p, 3p^2)$ on the curve $y = 3x^2$, where $p \neq 0$.
The normal to the curve at P meets the y-axis at the point Q.

Show that the y coordinate of Q is $3p^2 + \frac{1}{6}$.

$\frac{dy}{dx} = 6x$, so the gradient of the tangent at P is $6p$

Normal gradient $= -\frac{1}{6p}$

Normal: $y - 3p^2 = -\frac{1}{6p}(x - p)$

Put $x = 0$: $y = 3p^2 + \frac{p}{6p} = 3p^2 + \frac{1}{6}$

Answer: shown: $y = 3p^2 + \frac{1}{6}$

10 3D trigonometry: pyramid

VABCD is a pyramid. The base ABCD is a square with sides of 9 cm.
The vertex V is vertically above the centre O of the base. VA = 19 cm.
(a) Work out the vertical height VO.
(b) Work out the angle between VA and the base. (Calculator)

(a) AC = $9\sqrt{2}$ so AO = 6.364 cm

$VO^2 = 19^2 - 6.364^2 = 320.5$

VO = 17.9 cm

(b) $\cos VAO = \frac{AO}{VA} = \frac{6.364}{19}$, angle = 70.4°

Answer: (a) 17.9 cm (b) 70.4°