Day 78 **IGCSE 10-a-day: Model Solutions**

1 Tangent at a turning point

The graph of $y = 2x^2 - 4x + 2$ has a turning point at P.
Find the equation of the tangent to the curve at P.

$\frac{dy}{dx} = 4x - 4$

At P, $\frac{dy}{dx} = 0$, so $4x - 4 = 0$ and $x = 1$

$y = 0$ at P
The gradient is 0, so the tangent is horizontal: $y = 0$
Answer: $y = 0$

2 Radius from chord and height of arc

A circular arc has a chord AB of length 18 cm.
The greatest distance between the chord and the arc is 3 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so $OM = r - 3$
$AM = 18 \div 2 = 9$. Pythagoras in triangle OMA: $r^2 = (r - 3)^2 + 9^2$
$r^2 = r^2 - 6r + 9 + 81$
$6r = 90$, so $r = 15$ cm
Answer: 15 cm

3 Rearrange then differentiate

$y = \frac{5x^3 + 4x}{x}$

Find $\frac{dy}{dx}$.

Divide each term by x: $y = 5x^2 + 4$
$\frac{dy}{dx} = 10x$

Answer: $\frac{dy}{dx} = 10x$

4 Shaded areas: exact answers

ABCD is a square of side 12 cm.
BD is an arc of a circle with centre A and an arc of a circle with centre C.
Work out the area of the shaded region. Give your answer in terms of π.

Area of quarter circle (centre A, radius 12) = $\frac{1}{4} \times \pi \times 12^2 = 36\pi$

The two quarter circles overlap in the shaded region and together cover the whole square
Shaded area = $2 \times 36\pi - 144 = 72\pi - 144$
Answer: $(72\pi - 144)$ cm^2

5 Collinear points with vectors

$\mathbf{OA} = 4\mathbf{a} - \mathbf{b}$, $\mathbf{OB} = 6\mathbf{a} - 4\mathbf{b}$ and $\mathbf{OC} = 5\mathbf{b}$.
Show that A, B and C lie on a straight line.

$\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = 2\mathbf{a} - 3\mathbf{b}$
$\mathbf{AC} = \mathbf{OC} - \mathbf{OA} = -4\mathbf{a} + 6\mathbf{b}$
$\mathbf{AC} = (-2)\mathbf{AB}$
AC is a multiple of AB and they share the point A, so A, B and C are collinear
Answer: $\mathbf{AC} = -2\mathbf{AB}$, so A, B, C are collinear

6 Successive percentage change

A house price increases by 400%.
What percentage decrease would take it back to its original value?

Let the original be 100. After the increase it is 500
Decrease needed = 400
400 ÷ 500 × 100 = 80%
Answer: 80%

7 Vector of given length parallel to another

$\mathbf{p} = \begin{pmatrix} -7 \\ 24 \end{pmatrix}$.

Find a vector of length 75 that is parallel to $\mathbf{p}$.

$|\mathbf{p}| = \sqrt{(-7)^2 + (24)^2} = \sqrt{625} = 25$
Scale factor = 75 ÷ 25 = 3
$3\mathbf{p} = \begin{pmatrix} -21 \\ 72 \end{pmatrix}$
Answer: $\begin{pmatrix} \mathbf{-21} \\ \mathbf{72} \end{pmatrix}$ **(or** $\begin{pmatrix} \mathbf{21} \\ \mathbf{-72} \end{pmatrix}$**)**

8 Perpendicular bisector

A is the point (−2, −2) and B is the point (8, 2).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form y = mx + c.

Midpoint of AB = $(\frac{-2+8}{2}, \frac{-2+2}{2}) = (3, 0)$

Gradient of AB = $\frac{2-(-2)}{8-(-2)} = \frac{2}{5}$

The perpendicular gradient is $-1 \div \frac{2}{5} = -\frac{5}{2}$

Using the midpoint: $y = -\frac{5}{2}(x - 3)$

$y = -\frac{5}{2}x + \frac{15}{2}$

Answer: $\mathbf{y = -\frac{5}{2}x + \frac{15}{2}}$

9 Fractional indices

Given that x > 0, solve $x^{\frac{5}{4}} = 32$

Raise both sides to the power $\frac{4}{5}$

$x = (32)^{\frac{4}{5}}$
x = 16
Answer: x = 16

10 Wire cut in two

A piece of thin metal rod 28 cm long is cut into two pieces. One piece is bent to form a square of side x cm. The other piece is bent to form a rectangle whose length is three times its width.

(a) Show that the total area, A cm^2, enclosed by the two shapes is $A = x^2 + \frac{3(28 - 4x)^2}{64}$.

(b) Find the value of x for which A is least, and work out this least area.

Rectangle perimeter $28 - 4x = 8w$, so $w = \frac{28 - 4x}{8}$ and its area is $3w^2 = \frac{3(28 - 4x)^2}{64}$

$A = x^2 + \frac{3(28 - 4x)^2}{64} = \frac{7}{4}x^2 - \frac{21}{2}x + \frac{147}{4}$

$\frac{dA}{dx} = \frac{7}{2}x - \frac{21}{2} = 0$, so $x = 3$. $\frac{d^2A}{dx^2} = \frac{7}{2} > 0$, so a minimum

Square: 9, rectangle: $6 \times 2 = 12$, so $A = 21$

Answer: x = 3, least area 21 cm^2