Day 77 **IGCSE 10-a-day: Model Solutions**

1 f′(x) notation

f(x) = $5x^3 + 4x - 3$
(a) Find f′(x).
(b) Work out the value of f′(1).

(a) $5x^3 \rightarrow 15x^2$, $4x \rightarrow 4$
f′(x) = $15x^2 + 4$
(b) f′(1) = 19
Answer: (a) f′(x) = $15x^2 + 4$ (b) 19

2 Exact trigonometric values

Work out the exact value of cos 60° − cos 30°.
Give your answer in its simplest form, with no decimals.

cos 60° = $\frac{1}{2}$ and cos 30° = $\frac{\sqrt{3}}{2}$
Substitute and simplify (rationalise any denominator)
= $\frac{1-\sqrt{3}}{2}$
Answer: $\frac{\mathbf{1-\sqrt{3}}}{\mathbf{2}}$

3 Minimum gradient of a curve

A curve has equation $y = x^3 - 9x^2 + 29x + 3$.
Find the minimum value of the gradient of the curve, and the value of x at which it occurs.

$\frac{dy}{dx} = 3x^2 - 18x + 29$
The gradient is least when $\frac{d^2y}{dx^2} = 0$: $6x - 18 = 0$, so $x = 3$
$\frac{d^2y}{dx^2}$ increases through zero here (its derivative is 6 > 0), so this is a minimum
$\frac{dy}{dx} = 2$ when $x = 3$
Answer: minimum gradient 2 when x = 3

4 Direct variation

F is directly proportional to the cube of s.
When s = 6, F = 108.
(a) Find a formula for F in terms of s.
(b) Find the value of s when F = 171.5.

$F = ks^3$
$108 = k \times 6^3$ so $k = \frac{1}{2}$
$F = \frac{1}{2}s^3$
$171.5 = \frac{1}{2}s^3$ so $s^3 = 343$
$s = 7$
Answer: (a) $\mathbf{F = \frac{1}{2}s^3}$ **(b) s = 7**

5 Changing the subject

Make u the subject of $v = 2u^3 + 6$

Subtract 6: $v - 6 = 2u^3$

Divide by 2: $u^3 = \frac{v - 6}{2}$

Cube root: $u = \sqrt[3]{\frac{v - 6}{2}}$

Answer: $u = \sqrt[3]{\frac{v - 6}{2}}$

6 Area of a segment

The circle has centre O and radius 10 cm. A and B are points on the circle and angle AOB = 50°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{50}{360} \times \pi \times 10^2 = 43.633$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 10 \times 10 \times \sin 50° = 38.302$ cm^2

Segment = sector − triangle = 43.633 − 38.302 = 5.3310

= 5.33 cm^2 (3 s.f.)

Answer: 5.33 cm^2

7 Composite functions: fg and gf

$f(x) = 4x - 2$ and $g(x) = x^2 + 2$
Solve fg(x) = gf(x)

$fg(x) = f(x^2 + 2) = 4x^2 + 6$

$gf(x) = g(4x - 2) = (4x - 2)^2 + 2 = 16x^2 - 16x + 6$

Set them equal and rearrange: $12x^2 - 16x = 0$

Divide by 4 and factorise: $x(3x - 4) = 0$

$x = 0$ or $x = \frac{4}{3}$

Answer: $x = 0$ or $x = \frac{4}{3}$

8 Percentage change and proportion

The brightness of a lamp is inversely proportional to the square of the distance from the lamp.
The distance from the lamp is increased by 25%.
Work out the percentage change in the brightness of a lamp.

The distance from the lamp is multiplied by 1.25

The multiplier for the brightness of a lamp is $1 \div 1.25^2 = 0.64$

0.64 is a decrease of 36%

Answer: 36% decrease

9 Rectangle under a parabola

The diagram shows the curve $y = 150 - 2x^2$. Rectangle ABCD has A, B on the x-axis and C, D on the curve. B is (x, 0).
(a) Show that the area of ABCD is $-4x^3 + 300x$.
(b) Find the maximum area of ABCD, showing that it is a maximum.

AB = 2x and BC = $150 - 2x^2$, so area = $2x(150 - 2x^2) = -4x^3 + 300x$

$\frac{dA}{dx} = -12x^2 + 300 = 0$, so $x^2 = 25$ and $x = 5$ ($x > 0$)

$\frac{d^2A}{dx^2} = -24x < 0$ when $x = 5$, so a maximum

Area = $300 \times 5 - 4 \times 5^3 = 1000$

Answer: 1000

10 Nested surds

Show that $\sqrt{23 + 8\sqrt{7}} = 4 + \sqrt{7}$

Square the right-hand side: $(4 + \sqrt{7})^2 = 16 + 8\sqrt{7} + 7$

$= 23 + 8\sqrt{7}$

So $\sqrt{23 + 8\sqrt{7}} = 4 + \sqrt{7}$

Answer: $\sqrt{\mathbf{23} + \mathbf{8}\sqrt{7}} = \mathbf{4} + \sqrt{7}$