Day 76 **IGCSE 10-a-day: Model Solutions**

1 Point with a given gradient (quadratic)

A curve has equation $y = 2x^2 + 7x$.
The gradient at the point P on the curve is 3.
Find the coordinates of P.

$\frac{dy}{dx} = 4x + 7$
$4x + 7 = 3$, so $4x = -4$
$x = -1$
$y = -5$
Answer: $(-1, -5)$

2 Error intervals

The volume of juice is 3 litres, correct to the nearest whole number.
Write down the error interval for the volume of juice, using v.

The value has been rounded to the nearest 1
Half of this is 0.5, so go 0.5 below and 0.5 above
$3 - 0.5 = 2.5$ and $3 + 0.5 = 3.5$
$2.5 \leq v < 3.5$
Answer: $2.5 \leq v < 3.5$

3 Find unknown constants in a function

f(x) = ax + b, where a and b are constants.
$f(5) = 2$ and $f^{-1}(-10) = -1$.
Find the value of a and the value of b.

$f^{-1}(-10) = -1$ means $f(-1) = -10$
$5a + b = 2$ and $-a + b = -10$
Subtract: $6a = 12$, so $a = 2$
$b = 2 - 2 \times 5 = -8$
Answer: $a = 2, b = -8$

4 Exact trigonometric values

Work out the exact value of $\cos 45° \times \tan 60°$
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\cos 45° \times \tan 60° = \frac{\sqrt{2}}{2} \times \sqrt{3}$

$= \frac{\sqrt{6}}{2}$

Answer: $\frac{\sqrt{6}}{2}$

5 Trigonometric equations

Solve $4 \tan x° - 6 = -10$ for $0 \leq x \leq 360$

Rearrange: $4 \tan x° = -4$, so $\tan x° = -1$
The acute angle with tangent 1 is 45°; tangent is negative in the second and fourth quadrants
$x = 135$ or $x = 315$ $(180 - 45$ and $360 - 45)$
Answer: $x = 135, 315$

6 Minimum gradient of a curve

A curve has equation $y = 2x^3 + 12x^2 + 26x + 3$.
Find the minimum value of the gradient of the curve, and the value of x at which it occurs.

$\frac{dy}{dx} = 6x^2 + 24x + 26$

The gradient is least when $\frac{d^2y}{dx^2} = 0$: $12x + 24 = 0$, so $x = -2$

$\frac{d^2y}{dx^2}$ increases through zero here (its derivative is $12 > 0$), so this is a minimum

$\frac{dy}{dx} = 2$ when $x = -2$

Answer: minimum gradient 2 when $x = -2$

7 Solving trigonometric equations

Solve $2 \tan x = 0$ for $0 \leq x \leq 360$.

$\tan x = 0$

Principal value: $x = \tan^{-1}(0) = 0°$
Other solutions in the range: use symmetry of the tan graph
$x = 0, 180, 360$
Answer: $x = 0, 180, 360$

8 Gradient where the curve crosses the x-axis

A curve has equation $y = (x - 3)(x - 1)(x + 4)$.
Work out the gradient of the curve at each point where it crosses the x-axis.

Expand: $y = x^3 - 13x + 12$

$\frac{dy}{dx} = 3x^2 - 13$

The curve crosses the x-axis at $x = -4$, 1 and 3
Gradients: $x = -4$ gives 35, $x = 1$ gives -10, $x = 3$ gives 14
Answer: 35 at $x = -4$, -10 at $x = 1$, 14 at $x = 3$

9 Joint and inverse variation

P is directly proportional to the square of v and inversely proportional to t.
When $v = 5$ and $t = 3$, $P = 25$.
Work out the value of P when $v = 2$ and $t = 2$.

$P = \frac{kv^2}{t}$

$25 = \frac{k \times 25}{3}$, so $k = 25 \times 3 \div 25 = 3$

$P = \frac{3 \times 4}{2} = 6$

Answer: $P = 6$

10 Enlargement

Triangle P has vertices A(−2, −5), B(0, −3) and C(−2, −1).
Triangle Q has vertices A'(1, −4), B'(2, −3) and C'(1, −2).
Q is an enlargement of P.
Describe fully the single transformation that maps P onto Q.

x-step from A to B = 2, x-step from A' to B' = 1

Scale factor = $1 \div 2 = \frac{1}{2}$

The centre C satisfies: $C + \frac{1}{2}(A - C) = A'$, so $(1 - \frac{1}{2})C = A' - \frac{1}{2}A$

x: $\frac{1}{2} \times x = 1 - \frac{1}{2} \times (-2) = 2$, so $x = 4$

y: $\frac{1}{2} \times y = (-4) - \frac{1}{2} \times (-5) = -\frac{3}{2}$, so $y = -3$

Answer: Enlargement, scale factor $\frac{1}{2}$, centre (4, −3)