Day 75 **IGCSE 10-a-day: Model Solutions**

1 Stationary points of a cubic

A curve has equation $y = 2x^3 - 6x^2 - 18x + 2$.
Find the coordinates of the stationary points of the curve.

$\frac{dy}{dx} = 6x^2 - 12x - 18$

Stationary points: $\frac{dy}{dx} = 0$, so $6(x + 1)(x - 3) = 0$

$x = -1$ or $x = 3$
$y = 12$ when $x = -1$ and $y = -52$ when $x = 3$
Answer: $(-1, 12)$ and $(3, -52)$

2 Adding and scaling matrices

$A = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$ and $B = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$.
(a) Work out A + B.
(b) Work out 4A − 2B.

(a) $A + B = \begin{pmatrix} 11 \\ 3 \end{pmatrix}$

(b) $4A = \begin{pmatrix} 32 \\ 24 \end{pmatrix}$ and $2B = \begin{pmatrix} 6 \\ -6 \end{pmatrix}$

$4A - 2B = \begin{pmatrix} 26 \\ 30 \end{pmatrix}$

Answer: (a) $\begin{pmatrix} \mathbf{11} \\ \mathbf{3} \end{pmatrix}$ (b) $\begin{pmatrix} \mathbf{26} \\ \mathbf{30} \end{pmatrix}$

3 Square roots as powers

$y = -8x^{3/2} - \frac{8}{\sqrt{x}}$

Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Write as powers of x: $y = -8x^{3/2} - 8x^{-1/2}$
$-8x^{3/2} \rightarrow -12x^{1/2}$, $-8x^{-1/2} \rightarrow 4x^{-3/2}$
$\frac{dy}{dx} = -12x^{1/2} + 4x^{-3/2}$

Answer: $\frac{\mathbf{dy}}{\mathbf{dx}} = \mathbf{-12x^{1/2} + 4x^{-3/2}}$

4 Fractions of fractions

Uma spent $\frac{3}{7}$ of her money on a coat.

She then spent $\frac{7}{8}$ of the money that was left on a bag.

A bag cost £280.
How much money did Uma have to start with?

The second amount is $\frac{7}{8}$ of $\frac{4}{7}$, which is $\frac{1}{2}$ of the total

$\frac{1}{2}$ of the total is £280

Total = £280 ÷ $\frac{1}{2}$ = £560

Answer: £560

5 Venn diagram: reading set notation

The Venn diagram shows information about 27 visitors.
A: bought a drink B: bought a snack
(a) Find n(A′ ∪ B′).
(b) Find n(A′ ∩ B).

(a) n(A′ ∪ B′) = 8 + 12 + 1 = 21
(b) n(A′ ∩ B) = 12
Answer: (a) 21 (b) 12

6 Fencing against a wall

A farmer uses 108 m of fencing to enclose a rectangle against a wall, as shown.
(a) Show that the area, A m², is $A = 108x - 2x^2$.
(b) Find the maximum area, proving it is a maximum.

Side parallel to the wall = $108 - 2x$, so $A = x(108 - 2x) = -2x^2 + 108x$
$\frac{dA}{dx} = -4x + 108 = 0$ gives $x = 27$
$\frac{d^2A}{dx^2} = -4 < 0$, so a maximum
$A = 108 \times 27 - 2 \times 27^2 = 1458$
Answer: 1458 m²

7 At least one

The probability that a light bulb in a hallway fails in any month is $\frac{3}{10}$.
Events on different occasions are independent.
Work out the probability that this happens at least once in 3 months.
Give your answer correct to 3 significant figures. (Calculator)

$P(\text{never}) = (1 - \frac{3}{10})^3 = \frac{7}{10}^3 = 0.343$
P(at least once) = 1 − P(never) = 1 − 0.343
= 0.657
Answer: 0.657

8 Tangent to a circle: proof

Show that the line $y = -3x - 30$ is a tangent to the circle $x^2 + y^2 = 90$.
Find the coordinates of the point where the line touches the circle.

Substitute $y = -3x - 30$ into $x^2 + y^2 = 90$: $x^2 + (-3x - 30)^2 = 90$
Expand: $10x^2 + 180x + 810 = 0$
$b^2 - 4ac = 180^2 - 4 \times 10 \times 810 = 32400 - 32400 = 0$
$b^2 - 4ac = 0$, so there is exactly one point of intersection: the line is a tangent
$x = \frac{-180}{2 \times 10} = -9$, and $y = -3 \times (-9) - 30 = -3$. The point is $(-9, -3)$
Answer: (−9, −3)

9 Average speed

Grace drives 135 km at an average speed of 45 km/h.
The drive then continues for another 60 km at an average speed of 30 km/h.
Work out the average speed for the whole journey.

Time for first part = 135 ÷ 45 = 3 hours
Time for second part = 60 ÷ 30 = 2 hours
Total distance = 195 km, total time = 5 hours
Average speed = 195 ÷ 5 = 39 km/h
Answer: 39 km/h

10 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 9 cm, top radius 4 cm and base radius 16 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+9} = \frac{4}{16}$

16 h = 4(h + 9), so 12 h = 36 and h = 3

Large cone: $\frac{1}{3} \times \pi \times 16^2 \times 12 = 1024\pi$

Small cone: $\frac{1}{3} \times \pi \times 4^2 \times 3 = 16\pi$

Frustum = $1024\pi - 16\pi = 1008\pi$ cm^3

Answer: 1008π cm^3