Day 74 **IGCSE 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 06:05 (London time).
It takes 8 hours 30 minutes.
When it is 12:00 in London it is 21:00 in Sydney.
At what time, in Sydney time, does the flight land? Say if it is on a different day.

Arrival in London time = 06:05 + 8 h 30 min = 14:35
Sydney is 9 hours ahead of London, so add 9 hours
23:35
Answer: 23:35 the same day

2 Kinematics with roots

A particle moves in a straight line. After t seconds its displacement from O is s metres, where

$s = 24t^{\frac{3}{2}} + t^2$ $(t \geq 0)$
Find the velocity and the acceleration of the particle when t = 9.

$v = \frac{ds}{dt} = 2t + 36t^{\frac{1}{2}}$

$a = \frac{dv}{dt} = 2 + 18t^{-\frac{1}{2}}$

When t = 9: v = 126 and a = 8
Answer: v = 126 m/s, a = 8 m/s^2

3 Distance-time graphs

The graph shows Priya's cycle ride to a café and back.
(a) Work out Priya's speed on the way back.
(b) Work out Priya's average speed for the whole ride, including the stop.

(a) Return: 12 km in 1 hours, so speed = 12 ÷ 1 = 12 km/h
(b) Total distance = 2 × 12 = 24 km, total time = 3 hours
Average speed = 24 ÷ 3 = 8 km/h
Answer: (a) 12 km/h (b) 8 km/h

4 Estimating the mean from grouped data

The table shows the masses of 63 parcels.
(a) Write down the modal class.
(b) Work out an estimate for the mean. Give your answer to 1 decimal place. (Calculator)

(a) Highest frequency is 21: class $50 < x \leq 60$
(b) Midpoints: 25, 35, 45, 55
$\Sigma fx = 19 \times 25 + 4 \times 35 + 19 \times 45 + 21 \times 55 = 2625$
Mean $\approx 2625 \div 63 = 41.7$
Answer: (a) $50 < x \leq 60$ (b) 41.7

5 Show that the gradient is always positive

A curve has equation $y = 2x^3 + 3x^2 + 3x - 2$.
Show that y is an increasing function for all values of x.

$\frac{dy}{dx} = 6x^2 + 6x + 3$

Discriminant $= 6^2 - 4 \times 6 \times 3 = -36$

The discriminant is negative, so $\frac{dy}{dx} = 0$ has no real solutions

The coefficient of x^2 is positive, so $\frac{dy}{dx} > 0$ for all x

So y is increasing for all values of x
Answer: Discriminant = $-36 < 0$, so $dy/dx > 0$ for all x

6 Angles of elevation

A point A is on level ground in line with the base of a mast. (Calculator)
The angle of elevation of the top of the mast from A is 35°.
B is a point on the same line, 50 m further from the mast than A.
The angle of elevation of the top of the mast from B is 21°.
Work out the height of the mast. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 35° = \frac{h}{x}$, so $x = h \div \tan 35°$

From B: $\tan 21° = \frac{h}{x + 50}$, so $x + 50 = h \div \tan 21°$

Subtract: $50 = h \div \tan 21° - h \div \tan 35° = h(2.6051 - 1.4281)$
$h = 50 \div 1.1769 = 42.5$ m (1 d.p.)
Answer: 42.5 m

7 Open box of fixed surface area

An open container has a square base of side x cm and height h cm. The total outside surface area is 12 cm^2.

(a) Show that the volume of the box is $V = \frac{1}{4}(12x - x^3)$ cm^3.

(b) Find the maximum volume of the box, proving that it is a maximum.

$x^2 + 4xh = 12$, so $h = \frac{12 - x^2}{4x}$

$V = x^2h = \frac{x(12 - x^2)}{4} = \frac{1}{4}(12x - x^3)$

$\frac{dV}{dx} = -\frac{3}{4}x^2 + 3 = 0$, so $x^2 = 4$ and $x = 2$

$\frac{d^2V}{dx^2} = -\frac{3}{2}x = -3 < 0$, so a maximum

$V = 4$
Answer: 4 cm^3

8 Bounds: finding an unknown length

A rectangle has area 62 cm^2, correct to 2 significant figures.
Its length is 9.800000000000000710542735760100185871124267578125 cm, correct to 2 significant figures.
Work out the upper bound of the width of the rectangle. Give your answer correct to 3 significant figures. (Calculator)

Width = area ÷ length
Area: $61.5 \leq A < 62.5$. Length: $9.750000000000000710542735760100185871124267578125 \leq l < 9.850000000000000710542735760100185871124267578125$
UB width = UB area ÷ LB length = $62.5 \div 9.750000000000000710542735760100185871124267578125 = 6.41$
Answer: 6.41 cm

9 Loci and regions

ABCD is a rectangle with AB = 2 cm and AD = 7 cm.
A point P inside the rectangle is such that AP ≤ 2 cm and P is nearer to A than to B.
Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 1 cm from A
AP ≤ 2 cm: P is inside the quarter circle of radius 2 cm centred at A

The bisector meets the arc where $\cos\theta = \frac{1}{2}$, so $\theta = 60°$ at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 1 cm, height $2\sin 60° = 1\sqrt{3}$ cm, area $= \frac{1}{2} \times 1 \times 1\sqrt{3} = \frac{1}{2}\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 2^2 = \frac{1}{3}\pi$

Area $= \frac{1}{2}\sqrt{3} + \frac{1}{3}\pi$ cm^2

Answer: $\left(\frac{1}{2}\sqrt{3} + \frac{1}{3}\pi\right)$ cm^2

10 Tangency conditions

The line $y = -2x + k$ is a tangent to the curve $y = 3x^2 - 3x + 2$.
Find the value of k.

Equate: $3x^2 - 3x + 2 = -2x + k$

Rearrange: $3x^2 - x + 2 - k = 0$

A tangent touches at one point, so $b^2 - 4ac = 0$: $1 - 4 \times 3 \times (2 - k) = 0$

$12k - 23 = 0$

$k = \frac{23}{12}$

Answer: $k = \frac{23}{12}$