Day 73 **IGCSE 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 06:30 (London time).
It takes 13 hours 10 minutes.
When it is 12:00 in London it is 04:00 in Los Angeles.
At what time, in Los Angeles time, does the flight land? Say if it is on a different day.

Arrival in London time = 06:30 + 13 h 10 min = 19:40
Los Angeles is 8 hours behind London, so subtract 8 hours
11:40
Answer: 11:40 the same day

2 Magnitude of a vector

$\mathbf{v} = \begin{pmatrix} 7 \\ 24 \end{pmatrix}$.
Work out $|\mathbf{v}|$.

$|\mathbf{v}| = \sqrt{(7)^2 + (24)^2} = \sqrt{49 + 576}$
$= \sqrt{625} = 25$
Answer: 25

3 Averages with algebra

Here are five numbers: $x + 4$, $x + 6$, $x + 2$, $3x + 5$, $2x - 1$.
The mean of the five numbers is 16.
(a) Work out the value of x.
(b) Work out the range of the five numbers.

(a) Total = $(x + 4) + (x + 6) + (x + 2) + (3x + 5) + (2x - 1) = 16 \times 5 = 80$
$8x + 16 = 80$, so $8x = 64$ and $x = 8$
The numbers are 12, 14, 10, 29, 15
(b) Range = $29 - 10 = 19$
Answer: (a) x = 8 (b) 19

4 Particle at rest

A particle P moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where
$s = t^3 - 12t^2 + 36t + 6$ $(t \geq 0)$
Find the acceleration of P at each instant when it is at rest.

$v = \frac{ds}{dt} = 3t^2 - 24t + 36$

P is at rest when $v = 0$: $3t^2 - 24t + 36 = 0$
$3(t - 2)(t - 6) = 0$, so $t = 2$ or $t = 6$

$a = \frac{d^2s}{dt^2} = 6t - 24$

$t = 2$: $a = -12$. $t = 6$: $a = 12$
Answer: -12 m/s^2 when t = 2 and 12 m/s^2 when t = 6

5 Sample space and conditional probability

A spinner numbered 1 to 4 and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.
(a) Work out the probability that the sum is a multiple of 3.
(b) Given that at least one number is 3, work out the probability that the sum is a multiple of 3.

(a) There are $4 \times 5 = 20$ equally likely outcomes, and 7 of them satisfy: the sum is a multiple of 3

Probability = $\frac{7}{20}$

(b) 8 outcomes satisfy: at least one number is 3. Of these, 1 also satisfy: the sum is a multiple of 3

Probability = $\frac{1}{8}$

Answer: (a) $\frac{7}{20}$ (b) $\frac{1}{8}$

6 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 4 cm, DB = 8 cm and the area of triangle ADE is 16 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 4 + 8 = 12 cm
Length scale factor = 12 ÷ 4 = 3
Area scale factor = $(3)^2 = 9$
Area of triangle ABC = 16 × 9 = 144 cm^2
Trapezium = 144 − 16 = 128 cm^2
Answer: 128 cm^2

7 Same gradient on two curves

Two curves have equations $y = 4x^2 - 5x - 7$ and $y = 3x^2 - x + 9$.
Find the value of x at which the curves have the same gradient.

First curve: $\frac{dy}{dx} = 8x - 5$
Second curve: $\frac{dy}{dx} = 6x - 1$
Equal gradients: $8x - 5 = 6x - 1$
$2x = 4$, so $x = 2$
Answer: x = 2

8 Perpendicular bisector

A is the point (−7, −1) and B is the point (−5, −11).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = $(\frac{-7 + (-5)}{2}, \frac{-1 + (-11)}{2}) = (-6, -6)$
Gradient of AB = $\frac{-11 - (-1)}{-5 - (-7)} = -5$
The perpendicular gradient is $-1 \div (-5) = \frac{1}{5}$
Using the midpoint: $y + 6 = \frac{1}{5}(x + 6)$
Multiply by 5 and rearrange: $-x + 5y = -24$
Multiply by −1: $x - 5y = 24$
Answer: x − 5y = 24

9 Self-inverse function

$f(x) = \frac{4x - 4}{x - k}$, where k is a constant and $x \neq k$.

The function f is its own inverse, so $f^{-1}(x) = f(x)$.
Find the value of k.

Let $y = \frac{4x - 4}{x - k}$, so $y(x - k) = 4x - 4$
$xy - ky = 4x - 4$, so $x(y - 4) = ky - 4$
$f^{-1}(x) = \frac{kx - 4}{x - 4}$
For $f^{-1}(x) = f(x)$ the denominators must match: $x - 4 = x - k$
Answer: k = 4

10 Stationary point with a 1/x term

A curve has equation $y = 3x^2 + \frac{6}{x}$.

(a) Show that the curve has a stationary point at x = 1.

(b) Find the y coordinate of this point and decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = 6x - \frac{6}{x^2}$

When x = 1: $\frac{dy}{dx} = 6 - 6 = 0$

y = 9

$\frac{d^2y}{dx^2} = 6 + \frac{12}{x^3}$

At x = 1: $\frac{d^2y}{dx^2} = 18$, which is positive, so a minimum

Answer: y = 9, minimum