Day 72 **IGCSE 10-a-day: Model Solutions**

1 Gradient at a point

Work out the gradient of the curve $y = -3x^2 - 4x - 3$ at the point where $x = 0$.

$\frac{dy}{dx} = -6x - 4$

When $x = 0$: gradient $= -4$

Answer: -4

2 Index equations

Solve $9^{2x} = 27^2$

Write both sides as powers of 3: $9 = 3^2$ and $27 = 3^3$

Equate the powers: $4x = 6$

$4x = 6$, so $x = \frac{3}{2}$

Answer: $x = \frac{3}{2}$

3 Parallelogram and midpoint with vectors

ABCD is a parallelogram.

A = (4, 4), B = (3, 6) and D = (9, 3).

Find the coordinates of C and of the point where the diagonals meet.

$\mathbf{BC} = \mathbf{AD} = D - A = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

C = B + **AD** = (8, 5)

The diagonals bisect each other, so they meet at the midpoint of AC

$(\frac{4+8}{2}, \frac{4+5}{2}) = (6, \frac{9}{2})$

Answer: C = (8, 5), diagonals meet at $(6, \frac{9}{2})$

4 Exchange rates

Maya changes £1100 into euros at £1 = €1.2.

Maya spends €280 and then changes the rest back into pounds at £1 = €1.25.

How many pounds does Maya get back?

£1100 × 1.2 = €1320

1320 − 280 = €1040

1040 ÷ 1.25 = £832

Answer: £832

5 Curve touching the x-axis

The curve $y = -2x^3 - 6x^2 + k$, where k is a constant, has the x-axis as a tangent.

Find the two possible values of k.

$\frac{dy}{dx} = -6x^2 - 12x = 0$ gives $x = -2$ or $x = 0$

The x-axis is a tangent when a turning point has $y = 0$

$x = -2$: $-8 + k = 0$, so $k = 8$

$x = 0$: $0 + k = 0$, so $k = 0$

Answer: $k = 8$ or $k = 0$

6 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 6^2 + 9^2 = 117$
$AG^2 = AC^2 + CG^2 = 117 + 2^2 = 121$
AG = 11 cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{2}{\sqrt{117}}$

angle GAC = 10.5° (1 d.p.)
Answer: (a) 11 cm (b) 10.5°

7 Kinematics with roots

A particle moves in a straight line. After t seconds its displacement from O is s metres, where
$s = 24t^{\frac{3}{2}} + 2t^2$ $(t \geq 0)$
Find the velocity and the acceleration of the particle when t = 16.

$v = \frac{ds}{dt} = 4t + 36t^{\frac{1}{2}}$
$a = \frac{dv}{dt} = 4 + 18t^{-\frac{1}{2}}$

When t = 16: v = 208 and $a = \frac{17}{2}$

Answer: v = 208 m/s, a = $\frac{17}{2}$ m/s²

8 Iteration: rearranging equations

(a) Show that the equation $x^3 - 4x - 33 = 0$ can be rearranged to give $x = \sqrt[3]{4x + 33}$ (Calculator)
(b) Use the iteration formula $x_{n+1} = \sqrt[3]{4x_n + 33}$ with $x_0 = 3$ to work out x_1 and x_2. Give each answer correct to 3 decimal places.

(a) Rearrange $x^3 - 4x - 33 = 0$: $x^3 = 4x + 33$
Take the cube root of both sides: $x = \sqrt[3]{4x + 33}$
(b) $x_1 = \sqrt[3]{4 \times 3 + 33} = 3.55689...$
$x_2 = \sqrt[3]{4 \times x_1 + 33} = 3.61464...$
$x_1 = 3.557$ and $x_2 = 3.615$ (3 d.p.)
Answer: (b) $x_1 = 3.557$, $x_2 = 3.615$

9 Proof with indices

Show that $3^{t+2} + 3^t$ is always a multiple of 10.

$3^{t+2} = 3^t \times 3^2$
$3^{t+2} + 3^t = 3^t(3^2 + 1)$
$3^2 + 1 = 10$
$= 10 \times 3^t$, which is a multiple of 10
Answer: 10×3^t

10 Conditional probability from a two-way table

The table shows how 145 students choose a sport.

A person from the group "Year 10" is picked at random. Work out the probability that this person chose Hockey.

Total in the group "Year 10" = 64

Number who chose Hockey = 26

Probability = $\frac{26}{64} = \frac{13}{32}$

Answer: $\mathbf{\frac{13}{32}}$