Day 71 **IGCSE 10-a-day: Model Solutions**

1 Equations with fractions

Solve $\frac{x+6}{4} + \frac{x-6}{3} = -4$

Multiply every term by 12: $3(x + 6) + 4(x - 6) = -48$
Expand: $7x - 6 = -48$
$7x = -42$
$x = -6$
Answer: $x = -6$

2 Exact trigonometric values

Work out the exact value of $\tan 45° \times \sin 45°$
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\tan 45° \times \sin 45° = 1 \times \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{2}$

Answer: $\frac{\sqrt{2}}{2}$

3 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where
$s = 3t^2 + t + 6 + \frac{16}{t}$ $(t > 0)$
Find (a) the velocity of P, (b) the acceleration of P, when $t = 1$.

$v = \frac{ds}{dt} = 6t + 1 - 16t^{-2}$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 6 + 32t^{-3}$

When $t = 1$: $v = -9$ and $a = 38$
Answer: (a) -9 m/s (b) 38 m/s^2

4 Rational and irrational numbers

Here are six numbers.
$\sqrt{2} + \sqrt{8}$, $(\sqrt{6} + 2)(\sqrt{6} - 2)$, $\frac{\sqrt{18}}{\sqrt{2}}$, $\sqrt{8} \times \sqrt{2}$, $3 + \sqrt{12}$, $\frac{1}{\sqrt{3}}$
Write down the irrational numbers.

Simplify each number:

$\sqrt{2} + \sqrt{8} = 3\sqrt{2}$; $(\sqrt{6} + 2)(\sqrt{6} - 2) = 6 - 4 = 2$; $\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{9} = 3$

$\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$; $3 + \sqrt{12} = 3 + 2\sqrt{3}$; $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Answer: $\sqrt{2} + \sqrt{8}$, $3 + \sqrt{12}$, $\frac{1}{\sqrt{3}}$

5 Particle at rest

A particle P moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where
$s = 2t^3 - 18t^2 + 30t$ $(t \geq 0)$
Find the times when P is at rest.

$v = \frac{ds}{dt} = 6t^2 - 36t + 30$

P is at rest when $v = 0$: $6t^2 - 36t + 30 = 0$
$6(t - 1)(t - 5) = 0$, so $t = 1$ or $t = 5$
Answer: $t = 1$ and $t = 5$

6 Average speed

Nadia drives 45 km at an average speed of 30 km/h.
The drive then continues for another 35 km at an average speed of 35 km/h.
Work out the average speed for the whole journey.

Time for first part = 45 ÷ 30 = 1.5 hours
Time for second part = 35 ÷ 35 = 1 hour
Total distance = 80 km, total time = 2.5 hours
Average speed = 80 ÷ 2.5 = 32 km/h
Answer: 32 km/h

7 Exponential model y = ab^t

$y = ab^x$, where a and b are positive constants.
When x = 2, y = 2250. When x = 4, y = 20250.
Find the values of a and b, and the value of y when x = 6.

Divide: $b^2 = 20250 \div 2250 = 9$
b = 3
$a \times 3^2 = 2250$ so a = 250
$y = 250 \times 3^6 = 182250$
Answer: a = 250, b = 3, y = 182250

8 Tangent meets the axes

The diagram shows the curve $y = x^2 - 4x + 4$ and the tangent at P(4, 4), meeting the axes at A and B.
(a) Find an equation of the tangent at P.
(b) Work out the area of triangle OAB.

$\frac{dy}{dx} = 2x - 4$, so at x = 4 the gradient is 4
y − 4 = 4(x − 4), so y = 4x − 12
B: x = 0 gives y = −12. A: y = 0 gives x = 3
Area = ½ × 3 × 12 = 18
Answer: (a) y = 4x − 12 (b) 18

9 Combining means

The mean of the masses in kg for the boys is 46.
The mean of the masses in kg for the girls is 70.
The mean for all of the boys and girls together is 49.
Work out the ratio of the number of boys to the number of girls.

Let there be p boys and q girls
Total = 46p + 70q, and the overall mean gives 46p + 70q = 49(p + q)
−3p = −21q
p : q = −21 : −3 = 7 : 1
Answer: 7 : 1

10 Angles of elevation from two points

Points A and B are on level ground, 95 m apart, with B between A and the foot of a flagpole.
The angle of elevation of the top of the flagpole is 42° from A and 66° from B.
Work out the height of the flagpole. Give your answer to 3 significant figures. (Calculator)

Let the top be T. Angle ATB = 66° − 42° = 24° (exterior angle of the triangle)
Sine rule in triangle ABT: $\frac{AT}{\sin 114°} = \frac{95}{\sin 24°}$
$AT = \frac{95 \times \sin 66°}{\sin 24°} = 213.37$
Height = AT × sin 42° = 143 m
Answer: 143 m