Day 70 **IGCSE 10-a-day: Model Solutions**

1 nth term of a quadratic sequence

Here are the first five terms of a quadratic sequence.
13, 22, 35, 52, 73
Find an expression for the nth term.

First differences: 9, 13, 17, 21. Second differences: 4, 4, 4
Second difference = 2a, so a = 2
Subtract $2n^2$: 11, 14, 17, 20, 23
This is linear: 3n + 8
nth term = $2n^2 + 3n + 8$
Answer: $2n^2 + 3n + 8$

2 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = (3x − 15)°
Angle PQR = (3x − 20)°
Angle PRS = (4x + 1)°
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
4x + 1 = 6x − 35
2x = 36
x = 18
(b) Angle PRS = 73° so angle PRQ = 180° − 73° = 107°
Answer: (a) x = 18 (b) 107°

3 Percentage change

The price of a phone is increased by 15%.
The new price is then increased by 10%.
Work out the overall percentage change in the price.

Multipliers: 1.15 and 1.1
Overall multiplier = 1.15 × 1.1 = 1.265
1.265 − 1 = 0.265, so an overall increase of 26.5%
Answer: 26.5% increase

4 Trigonometric equations

Solve 3 tan x° − 5 = −8 for $0 \leq x \leq 360$

Rearrange: 3 tan x° = −3, so tan x° = −1
The acute angle with tangent 1 is 45°; tangent is negative in the second and fourth quadrants
x = 135 or x = 315 (180 − 45 and 360 − 45)
Answer: x = 135, 315

5 Error intervals

The height of a tower is 1.7 m, truncated to 1 decimal place.
Write down the error interval for the height of a tower, using h.

Truncating cuts off the later digits without rounding.
The smallest possible value is 1.7 and the value must be less than 1.8
$1.7 \leq h < 1.8$
Answer: $1.7 \leq h < 1.8$

6 Find constants from a turning point

The curve $y = x^3 + ax^2 + bx$ has stationary points at $x = -1$ and $x = 1$.
Find the values of a and b.

$\frac{dy}{dx} = 3x^2 + 2ax + b$

$x = -1$: $3 + -2a + b = 0$. $x = 1$: $3 + 2a + b = 0$

Subtract the equations: $0 + -4a = 0$, so $a = 0$

Then $b = -3$

Answer: a = 0, b = −3

7 Inverse proportion

y is inversely proportional to x.
When x = 5, y = 4.
Find the value of y when x = 10.

$y = \frac{k}{x}$

$4 = \frac{k}{5}$, so $k = 4 \times 5 = 20$

$y = \frac{20}{10} = 2$

Answer: y = 2

8 Fastest rate of growth

The number of bacteria in a culture, N, after t hours is modelled by
$N = -t^3 + 15t^2 + 2t + 500$ for $0 \leq t \leq 20$
Find the time at which N is increasing most rapidly, and the rate of increase at this time.

$\frac{dN}{dt} = -3t^2 + 30t + 2$

Rate is greatest when $\frac{d^2N}{dt^2} = -6t + 30 = 0$, so $t = 5$

$\frac{d^2N}{dt^2}$ is decreasing (gradient −6), so the rate is a maximum

$\frac{dN}{dt} = 77$ when $t = 5$

Answer: t = 5, rate 77 per hour

9 Probability and quadratic equations

A bag contains x white beads and 3 red beads.
Two beads are taken at random without replacement.
The probability that both are white is $\frac{6}{11}$.

(a) Show that $5x^2 - 41x - 36 = 0$.
(b) Hence work out the number of beads in the bag.

(a) P(both white) $= \frac{x}{x+3} \times \frac{x-1}{x+2} = \frac{6}{11}$

$11\,x(x - 1) = 6(x + 3)(x + 2)$

$11x^2 - 11x = 6x^2 + 30x + 36$ so $5x^2 - 41x - 36 = 0$

(b) Solve: $x = 9$ or $x = -\frac{4}{5}$

x cannot be negative, so $x = 9$

Number of beads = 9 + 3 = 12

Answer: x = 9, so 12 beads in the bag

10 Tangent meets the curve again

A curve has equation $y = x^3 + 5x^2 + 9x - 3$.
The tangent to the curve at the point A, where $x = -3$, meets the curve again at the point B.
Find the coordinates of B.

$\frac{dy}{dx} = 3x^2 + 10x + 9$, so the gradient at A is 6, and A = $(-3, -12)$

Tangent: $y = 6x + 6$

Equate: $x^3 + 5x^2 + 3x - 9 = 0$, which has a repeated root $x = -3$

Sum of the roots = -5, so $-3 + (-3) + x = -5$ and $x = 1$

$y = 12$

Answer: B (1, 12)