Day 7 **IGCSE 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 15:15 (London time).
It takes 12 hours 35 minutes.
When it is 12:00 in London it is 14:00 in Cairo.
At what time, in Cairo time, does the flight land? Say if it is on a different day.

Arrival in London time = 15:15 + 12 h 35 min = 03:50 the next day
Cairo is 2 hours ahead of London, so add 2 hours
05:50 the next day
Answer: 05:50 the next day

2 Second derivative and nature at a given point

$y = 2x^3 - 24x + 8$
Use the second derivative to show that the curve has a maximum at $x = -2$.

$\frac{dy}{dx} = 6x^2 - 24$

When $x = -2$: $\frac{dy}{dx} = 0$, so it is a stationary point

$\frac{d^2y}{dx^2} = 12x$

When $x = -2$: $\frac{d^2y}{dx^2} = -24$, which is negative

So there is a maximum at $x = -2$
Answer: maximum at $x = -2$

3 Parallelogram and midpoint with vectors

ABCD is a parallelogram.
A = (0, −2), B = (1, 2) and D = (0, 1).
Find the coordinates of C and of the point where the diagonals meet.

$\mathbf{BC} = \mathbf{AD} = D - A = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

C = B + **AD** = (1, 5)
The diagonals bisect each other, so they meet at the midpoint of AC
$(\frac{0+1}{2}, \frac{-2+5}{2}) = (\frac{1}{2}, \frac{3}{2})$

Answer: C = (1, 5), diagonals meet at $(\frac{1}{2}, \frac{3}{2})$

4 Increasing or decreasing at a point

$y = 2x^3 - 3x^2 - 7$
At the point where x = 3, is the curve increasing or decreasing? You must show your working.

$\frac{dy}{dx} = 6x^2 - 6x$

When $x = 3$: $\frac{dy}{dx} = 36$

36 is positive, so the curve is increasing
Answer: increasing (gradient 36)

5 Arc length and sector area

A sector of a circle has radius 10 cm and angle 240°.
(a) Work out the length of the arc of the sector.
(b) Work out the area of the sector. Give each answer in terms of π.

(a) Fraction of the circle = $\frac{240}{360}$

Arc length = $\frac{240}{360} \times 2 \times \pi \times 10 = \frac{40}{3}\pi$ cm

(b) Area = $\frac{240}{360} \times \pi \times 10^2 = \frac{200}{3}\pi$ cm^2

Answer: (a) $\frac{40}{3}\pi$ cm (b) $\frac{200}{3}\pi$ cm^2

6 Completing the square

(a) Write $3x^2 - 18x + 28$ in the form $a(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = 3x^2 - 18x + 28$

(a) Take out the factor 3 from the x terms: $3(x^2 - 6x) + 28$
$x^2 - 6x = (x - 3)^2 - 9$, so $3x^2 - 18x + 28 = 3[(x - 3)^2 - 9] + 28$
$= 3(x - 3)^2 + 1$
(b) The minimum is when $x - 3 = 0$, so $x = 3$ and $y = 1$
Answer: (a) $3(x - 3)^2 + 1$ (b) (3, 1)

7 Three sets from information

51 members of a gym were asked which activities they do.
A: use the pool B: use the sauna C: use the studio
$n(A) = 33$, $n(B) = 38$, $n(C) = 25$, $n(A \cap B) = 25$, $n(A \cap C) = 19$, $n(B \cap C) = 17$, $n(A \cap B \cap C) = 14$.
(a) Work out how many are in none of A, B and C.
(b) Work out how many are in exactly one of A, B and C.

$n(A \cup B \cup C) = 33 + 38 + 25 - 25 - 19 - 17 + 14 = 49$
(a) Number in none of the sets = $51 - 49 = 2$
(b) Only A = $33 - 25 - 19 + 14 = 3$
Only B = 10, only C = 3, so exactly one = $3 + 10 + 3 = 16$
Answer: (a) 2 (b) 16

8 Kinematics with roots

A particle moves in a straight line. After t seconds its displacement from O is s metres, where
$s = 16t^{\frac{3}{2}} + 3t^2$ $(t \geq 0)$
Find the velocity and the acceleration of the particle when t = 4.

$v = \frac{ds}{dt} = 6t + 24t^{\frac{1}{2}}$

$a = \frac{dv}{dt} = 6 + 12t^{-\frac{1}{2}}$

When t = 4: v = 72 and a = 12
Answer: v = 72 m/s, a = 12 m/s^2

9 Percentage change and proportion

The brightness of a lamp is inversely proportional to the square of the distance from the lamp.
The distance from the lamp is increased by 300%.
Work out the percentage change in the brightness of a lamp.

The distance from the lamp is multiplied by 4
The multiplier for the brightness of a lamp is $1 \div 4^2 = 0.0625$
0.0625 is a decrease of 93.75%
Answer: 93.75% decrease

10 Perpendicular bisector

A is the point (4, 4) and B is the point (10, 0).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form y = mx + c.

Midpoint of AB = $(\frac{4+10}{2}, \frac{4+0}{2}) = (7, 2)$

Gradient of AB = $\frac{0-4}{10-4} = -\frac{2}{3}$

The perpendicular gradient is $-1 \div (-\frac{2}{3}) = \frac{3}{2}$

Using the midpoint: $y - 2 = \frac{3}{2}(x - 7)$

$y = \frac{3}{2}x - \frac{17}{2}$

Answer: $y = \frac{3}{2}x - \frac{17}{2}$