Day 69 **IGCSE 10-a-day: Model Solutions**

1 Negative powers

$y = -6x^3 - \frac{5}{x^2}$

Find $\frac{dy}{dx}$.

Write negative powers as indices: $y = -6x^3 - 5x^{-2}$

$-6x^3 \to -18x^2$, $-5x^{-2} \to 10x^{-3}$

$\frac{dy}{dx} = -18x^2 + 10x^{-3}$

Answer: $\mathbf{\frac{dy}{dx} = -18x^2 + \frac{10}{x^3}}$

2 Finding a percentage rate

A machine was bought for £14,500.
Each year its value decreases by the same percentage.
After 3 years it is worth £4,973.50.
Work out the percentage decrease each year.

Multiplier for 3 years = £4,973.50 ÷ £14,500 = 0.343
Yearly multiplier = the cube root of 0.343 = 0.7
30% decrease each year
Answer: 30%

3 Line and curve intersections

Solve the simultaneous equations
$y = x^2 + 4x + 4$
$y = -x$

Substitute: $x^2 + 4x + 4 = -x$
Rearrange: $x^2 + 5x + 4 = 0$
Factorise: $(x + 1)(x + 4) = 0$, so $x = -1$ or $x = -4$
When $x = -1$: $y = -1 \times (-1) = 1$
When $x = -4$: $y = -1 \times (-4) = 4$
Answer: $x = -4$, $y = 4$ and $x = -1$, $y = 1$

4 Turning point of a quadratic by differentiation

$f(x) = x^2 - 8$
Use differentiation to find the coordinates of the turning point of the graph of $y = f(x)$. State whether it is a maximum or a minimum.

$f'(x) = 2x$
$f'(x) = 0$: $2x = 0$, so $x = 0$
$f(0) = -8$
$f''(x) = 2$, which is positive, so it is a minimum
Answer: (0, −8), minimum

5 Venn diagram: shaded region notation

Write down the set notation for the region shaded in the Venn diagram.

Identify which regions are shaded, using the circles E and F
The shaded region is $F \cup E'$
Answer: $F \cup E'$

6 Minimum point of y = ax + b/x

The diagram shows a sketch of the curve $y = 3x + \frac{48}{x}$ for $x > 0$.

The curve has a minimum point at M.

Find the coordinates of M, justifying that it is a minimum.

$y = 3x + 48x^{-1}$, so $\frac{dy}{dx} = 3 - 48x^{-2}$

$\frac{dy}{dx} = 0$: $3 = \frac{48}{x^2}$, so $x^2 = 16$ and $x = 4$ $(x > 0)$

$\frac{d^2y}{dx^2} = 96x^{-3} = \frac{3}{2} > 0$, so a minimum

$y = 12 + \frac{48}{4} = 24$

Answer: M (4, 24)

7 Show that: trigonometric identities

Show that $(3 \sin \theta + 2 \cos \theta)^2 + (2 \sin \theta - 3 \cos \theta)^2 \equiv 13$.

First bracket: $9 \sin^2 \theta + 12 \sin \theta \cos \theta + 4 \cos^2 \theta$

Second bracket: $4 \sin^2 \theta - 12 \sin \theta \cos \theta + 9 \cos^2 \theta$

Add: the $\sin \theta \cos \theta$ terms cancel, leaving $13 \sin^2 \theta + 13 \cos^2 \theta$

$= 13(\sin^2 \theta + \cos^2 \theta) = 13$

Answer: Shown

8 Cumulative frequency graphs

The cumulative frequency graph gives information about the marks of 100 students in a test.

(a) Find an estimate for the median and the interquartile range.

(b) Estimate how many of the 100 had a mark greater than 25.

(a) Median at cumulative frequency 50, so median = 50

Lower quartile at 25, so LQ = 40. Upper quartile at 75, so UQ = 60

Interquartile range = 60 − 40 = 20

(b) At 25, cumulative frequency = 11

Number greater = 100 − 11 = 89

Answer: (a) median 50, IQR 20 (b) 89

9 Tangent to a circle

A circle has equation $x^2 + y^2 = 841$.

The point P(−20, 21) lies on the circle.

(a) Find the equation of the tangent to the circle at P. Give your answer in the form $ax + by = c$, where a, b and c are integers.

(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $-\frac{21}{20}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div (-\frac{21}{20}) = \frac{20}{21}$

$y - 21 = \frac{20}{21}(x - (-20))$

Multiply out and rearrange: $20x - 21y = -841$

(b) On the x-axis $y = 0$, so $20x = -841$

$x = -\frac{841}{20}$, so T is $(-\frac{841}{20}, 0)$

Answer: (a) $20x - 21y = -841$ (b) T = $(-\frac{841}{20}, 0)$

10 Bounds of roots

The area of a square is 148 cm^2, correct to the nearest cm^2.
Work out the upper bound of the side length of the square.
Give your answer correct to 3 significant figures. (Calculator)

$147.5 \leq$ area < 148.5
Upper bound of side = $\sqrt{148.5}$
= 12.1861... = 12.2 (3 s.f.)
Answer: 12.2 cm