Day 68 **IGCSE 10-a-day: Model Solutions**

1 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where $s = 3t^3 - 2t^2 + 3t + 4$ $(t > 0)$
Find (a) the velocity of P, (b) the acceleration of P, when t = 3.

$v = \frac{ds}{dt} = 9t^2 - 4t + 3$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 18t - 4$

When t = 3: v = 72 and a = 50

Answer: (a) 72 m/s (b) 50 m/s^2

2 Weighted percentages

The ratio of Year 10 students to Year 11 students in a school is 1 : 4.
30% of the Year 10 students and 85% of the Year 11 students have a part-time job.
What percentage of all of these students have a part-time job?

Imagine 100 students in total: 20 Year 10 students and 80 Year 11 students
30% of 20 = 6 and 85% of 80 = 68
Together: 74 out of 100
74 ÷ 100 × 100 = 74%

Answer: 74%

3 HCF and LCM

(a) Write 1512 and 3024 as products of prime factors.
(b) Find the HCF and the LCM of 1512 and 3024.
Give your answers as numbers.

(a) $1512 = 2^3 \times 3^3 \times 7$ and $3024 = 2^4 \times 3^3 \times 7$

(b) HCF = $2^3 \times 3^3 \times 7 = 1512$

LCM = $2^4 \times 3^3 \times 7 = 3024$

Answer: (a) $2^3 \times 3^3 \times 7$; $2^4 \times 3^3 \times 7$ (b) HCF = 1512, LCM = 3024

4 Tangent where the curve crosses the y-axis

The curve $y = x^3 + x^2 - x - 3$ crosses the y-axis at the point A.
The tangent to the curve at A meets the x-axis at B.
Find the coordinates of B.

At A, x = 0 and y = −3

$\frac{dy}{dx} = 3x^2 + 2x - 1$, so the gradient at A is −1

Tangent: $y = -x - 3$

At B, y = 0: $-x - 3 = 0$, so x = −3

Answer: (−3, 0)

5 3D trigonometry: cuboid

ABCDEFGH is a cuboid. AB = 11 cm, BC = 11 cm and CG = 3 cm.
(a) Work out the length of AG.
(b) Work out the size of angle GAB.
Give answers to 3 significant figures or 1 decimal place. (Calculator)

(a) $AG^2 = 11^2 + 11^2 + 3^2 = 251$, so AG = 15.84 cm

(b) $BG^2 = 11^2 + 3^2 = 130$. AB is perpendicular to the plane BCGF, so angle ABG = 90°

Triangle ABG is right-angled at B, so $\cos GAB = \frac{AB}{AG} = \frac{11}{15.84}$

Angle GAB = 46.0°

Answer: (a) 15.8 cm (b) 46.0°

6 Nature from the gradient either side

$y = 2x^3 - 3x^2 + 2$

(a) Show that there is a stationary point at $x = 1$.

(b) By finding the gradient when $x = \frac{1}{2}$ and when $x = \frac{3}{2}$, decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = 6x^2 - 6x$, so when $x = 1$: $\frac{dy}{dx} = 0$

$x = \frac{1}{2}$: gradient $= -\frac{3}{2}$ (negative)

$x = \frac{3}{2}$: gradient $= \frac{9}{2}$ (positive)

The gradient goes from negative to 0 to positive, so (1, 1) is a minimum

Answer: (b) minimum

7 Transformations of graphs

The graph of $y = f(x)$ has a maximum point at (4, 5).

Write down the coordinates of the turning point of the graph of

(a) $y = 3f(x)$

(b) $y = f(x - 1) + 5$

(a) Start with the turning point (4, 5)

Apply the transformation to the point: (4, 15)

(b) $f(x - 1)$ changes the x-coordinate to 5, and $f(x) + 5$ changes the y-coordinate to 10

Turning point: (5, 10)

Answer: (a) (4, 15) (b) (5, 10)

8 Arithmetic sequence from two terms

An arithmetic sequence has term 2 equal to -24 and term 5 equal to -42.

(a) Find an expression for the nth term.

(b) Find the first term of the sequence that is less than -80.

(a) Common difference $= (-42 - (-24)) \div 3 = -6$

nth term $= -6n + c$; $-24 = (-6) \times 2 + c$ so $c = -12$

nth term $= -6n - 12$

(b) $-6n - 12 < -80$: $n < \frac{34}{3}$

Smallest whole number $n = 12$; the term is -84

Answer: (a) $-6n - 12$ (b) -84

9 Vector proof: collinear points

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

$\overrightarrow{OP} = 3\mathbf{a} + \mathbf{b}$, $\overrightarrow{OQ} = -3\mathbf{a} - 3\mathbf{b}$ and $\overrightarrow{OR} = -12\mathbf{a} - 9\mathbf{b}$.

(a) Show that P, Q and R lie on a straight line.

(b) Work out the ratio PQ : QR in its simplest form.

(a) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = -6\mathbf{a} - 4\mathbf{b}$

$\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ} = -9\mathbf{a} - 6\mathbf{b}$

$\overrightarrow{QR} = \frac{3}{2}\overrightarrow{PQ}$, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line

(b) PQ : QR = 2 : 3

Answer: (b) 2 : 3

10 Equal roots and real roots

The equation $x^2 + 6x + k = 0$ has two different real solutions.
Find the range of possible values of k.

Two different real solutions means $b^2 - 4ac > 0$

$6^2 - 4 \times 1 \times k > 0$, so $36 - 4k > 0$

$36 > 4k$, so $k < 9$

Answer: $k < 9$