Day 67 **IGCSE 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of members in each of four groups.

(a) A stratified sample of 27 is taken. Work out how many come from Group C.

(b) A different stratified sample has 12 from Group C. Work out the size of this sample.

Total = 175 + 175 + 150 + 175 = 675

(a) Group C: $\frac{150}{675} \times 27 = 6$

(b) Fraction sampled = 12 ÷ 150 = $\frac{2}{25}$, so sample size = $\frac{2}{25} \times 675 = 54$

Answer: (a) 6 (b) 54

2 Quadratic formula and surds

Solve $x^2 + 4x - 8 = 0$

Give your answers in the form $a \pm b\sqrt{c}$

Complete the square: $(x + 2)^2 - 4 - 8 = 0$

$(x + 2)^2 = 12$

$x + 2 = \pm \sqrt{12} = \pm 2\sqrt{3}$

$x = -2 \pm 2\sqrt{3}$

Answer: $x = -2 \pm 2\sqrt{3}$

3 Rearrange then differentiate

$y = \frac{(5x + 2)^2}{x}$

Find $\frac{dy}{dx}$.

Expand the bracket: $y = \frac{25x^2 + 20x + 4}{x} = 25x + 20 + 4x^{-1}$

$25x \to 25$, $4x^{-1} \to -4x^{-2}$

$\frac{dy}{dx} = 25 - 4x^{-2}$

Answer: $\frac{dy}{dx} = 25 - \frac{4}{x^2}$

4 Multiplying and dividing surds

Simplify fully $\frac{3\sqrt{75}}{2\sqrt{3}}$

$\sqrt{75} = 5\sqrt{3}$

$\frac{3\sqrt{75}}{2\sqrt{3}} = \frac{15\sqrt{3}}{2\sqrt{3}}$

$= \frac{15}{2}$

Answer: $\frac{15}{2}$

5 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 4. N is on AB with AN : NB = 1 : 2.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find MN in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{1}{3}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a}) = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

(b) OM = $\frac{1}{5}\mathbf{a}$

MN = ON − OM = $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} - \frac{1}{5}\mathbf{a} = \frac{7}{15}\mathbf{a} + \frac{1}{3}\mathbf{b}$

Answer: (a) $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ (b) $\frac{7}{15}\mathbf{a} + \frac{1}{3}\mathbf{b}$

6 Depreciation

A van is bought for £18,000.
Its value falls by 8% each year.
Work out its value after 5 years. (Calculator)

Multiplier = 1 − 8 ÷ 100 = 0.92
Value = 18000×0.92^5
= £11,863.47
Answer: £11,863.47

7 Find constants from a turning point

The diagram shows a sketch of the curve $y = ax^2 + bx + 19$.
The curve has a turning point at M(3, 1).
Find the values of a and b.

$\frac{dy}{dx} = 2ax + b = 0$ when x = 3, so $b = -6a$
The point (3, 1) is on the curve: $9a + 3b + 19 = 1$
Substitute $b = -6a$: $-9a = -18$, so $a = 2$ and $b = -12$
Answer: $a = 2$, $b = -12$

8 Inclusion and exclusion with two sets

There are 79 visitors.
15 of them bought a drink and 15 of them bought a snack.
57 of them do neither.
(a) Work out how many do both.
(b) One of the visitors is chosen at random. Find the probability that they only bought a drink.

Number in A or B = 79 − 57 = 22
(a) $n(A \cap B) = n(A) + n(B) - n(A \cup B) = 15 + 15 - 22 = 8$
(b) Number who only bought a drink = 15 − 8 = 7

Probability = $\frac{7}{79}$

Answer: (a) 8 (b) $\frac{7}{79}$

9 Finding the rate of interest

Maya invested £12,500 in a savings account for 3 years.
Compound interest was paid at a fixed rate per year.
At the end of the 3 years the account held £14,060.80.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $12500 \times m^3 = 14060.80$

$m^3 = 14060.80 \div 12500 = 1.124864$

$m = \sqrt[3]{1.124864} = 1.04$

Rate $= (1.04 - 1) \times 100 = 4\%$

Answer: 4% per year

10 Gradient where the curve crosses the x-axis

A curve has equation $y = (x + 3)(x - 3)(x + 2)$.
Work out the gradient of the curve at each point where it crosses the x-axis.

Expand: $y = x^3 + 2x^2 - 9x - 18$

$\frac{dy}{dx} = 3x^2 + 4x - 9$

The curve crosses the x-axis at $x = -3$, -2 and 3

Gradients: $x = -3$ gives 6, $x = -2$ gives -5, $x = 3$ gives 30

Answer: 6 at $x = -3$, -5 at $x = -2$, 30 at $x = 3$