Day 66 **IGCSE 10-a-day: Model Solutions**

1 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where $s = 2t^3 + 6t^2 + 5t + 1$ $(t > 0)$

Find (a) the velocity of P, (b) the acceleration of P, when t = 3.

$v = \frac{ds}{dt} = 6t^2 + 12t + 5$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 12t + 12$

When t = 3: v = 95 and a = 48

Answer: (a) 95 m/s (b) 48 m/s^2

2 Inverse of a linear function

$f(x) = -4x - 3$.

(a) Find $f^{-1}(x)$.

(b) Find $f^{-1}(-1)$.

(a) Let $y = -4x - 3$, then rearrange to make x the subject

$x = \frac{-3 - y}{4}$, so $f^{-1}(x) = \frac{-3 - x}{4}$

(b) $f^{-1}(-1) = -\frac{1}{2}$

Answer: (a) $\mathbf{f^{-1}(x) = \frac{-3 - x}{4}}$ (b) $\mathbf{-\frac{1}{2}}$

3 Rearrange then differentiate

$y = \frac{3x^3 - 2x}{x}$

Find $\frac{dy}{dx}$.

Divide each term by x: $y = 3x^2 - 2$

$\frac{dy}{dx} = 6x$

Answer: $\mathbf{\frac{dy}{dx} = 6x}$

4 Using a line of best fit

The scatter graph shows age of car (x) and value (y).

The equation of the line of best fit is $y = -0.5x + 32$.

(a) Estimate y when x = 3.

(b) Explain why an estimate of y when x = 15 may be unreliable.

(a) $y = -0.5 \times 3 + 32 = 30.5$

(b) x = 15 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)

Answer: (a) 30.5 (b) x = 15 is outside the range of the data

5 Percentages

B is 5% less than Q.

Q is 30% more than P.

Work out B as a percentage of P.

$B = 0.95 \times Q$ and $Q = 1.3 \times P$

$B = 0.95 \times 1.3 \times P = 1.235 \times P$

$1.235 \times 100 = 123.5\%$

Answer: 123.5%

6 Area and quadratic equations

The diagram shows a rectangle of length $(x - 1)$ cm, width $(x - 3)$ cm and area 15 cm^2.
(a) Show that $x^2 - 4x - 12 = 0$.
(b) Work out the length of the diagonal of the rectangle. Give your answer correct to 3 significant figures.

(a) $(x - 1)(x - 3) = 15$
Expand: $x^2 - 4x + 3 = 15$, so $x^2 - 4x - 12 = 0$
(b) Factorise: $(x - 6)(x + 2) = 0$
$x = 6$ ($x = -2$ is not possible as a length would be negative)
Length = 5 cm and width = 3 cm
Diagonal = $\sqrt{5^2 + 3^2} = \sqrt{34} = 5.83$ cm (3 s.f.)
Answer: (b) 5.83 cm

7 Area of triangle from a tangent

The tangent to the curve $y = 3x^2 - 5x - 5$ at the point where $x = -2$ meets the axes at A and B.
Work out the area of triangle OAB, where O is the origin.

$y = 17$ and $\frac{dy}{dx} = 6x - 5$, so the gradient is -17
Tangent: $y = -17x - 17$
y-axis: $(0, -17)$. x-axis: $0 = -17x - 17$ so $x = -1$
Area $= \frac{1}{2} \times 17 \times 1$
$= \frac{17}{2}$
Answer: $\frac{17}{2}$ square units

8 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 2 : 3. N is on OB with ON : NB = 1 : 1. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM $= \frac{2}{5}\mathbf{a}$, so BM $= -\mathbf{b} + \frac{2}{5}\mathbf{a} = \frac{2}{5}\mathbf{a} - \mathbf{b}$
(b) X is on BM: BX = s BM, so OX = OB + s BM $= \frac{2}{5}s\mathbf{a} + (1 - s)\mathbf{b}$
X is on AN: OX = OA + t AN = OA + t(ON − OA) $= (1 - t)\mathbf{a} + \frac{1}{2}t\mathbf{b}$
Equate: $\frac{2}{5}s = 1 - t$ and $1 - s = \frac{1}{2}t$
Solving: $s = \frac{5}{8}$ and $t = \frac{3}{4}$
OX $= \frac{1}{4}\mathbf{a} + \frac{3}{8}\mathbf{b}$
Answer: (a) $\frac{2}{5}\mathbf{a} - \mathbf{b}$ (b) OX $= \frac{1}{4}\mathbf{a} + \frac{3}{8}\mathbf{b}$

9 A squared in terms of A and I

$A = \begin{pmatrix} 4 & -2 \\ 6 & 5 \end{pmatrix}$.

$A^2 = pA + qI$, where I is the 2×2 identity matrix.
Find the value of p and the value of q.

$A^2 = \begin{pmatrix} 4 & -18 \\ 54 & 13 \end{pmatrix}$

Top right entries: $-18 = p \times (-2)$, so $p = 9$
Top left entries: $4 = 9 \times 4 + q$, so $q = -32$
Check bottom right: $9 \times 5 - 32 = 13$
Answer: $p = 9$, $q = -32$

10 Equations in disguise

Solve $x - 6\sqrt{x} + 8 = 0$

Let $u = \sqrt{x}$: $u^2 - 6u + 8 = 0$
Factorise: $(u - 4)(u - 2) = 0$, so $u = 4$ or $u = 2$
$\sqrt{x} = 4$ or $\sqrt{x} = 2$
$x = 16$ or $x = 4$
Answer: $x = 4$ or $x = 16$