Day 65 **IGCSE 10-a-day: Model Solutions**

1 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where $s = -t^3 + 6t^2 - 5t$ $(t > 0)$

Find (a) the velocity of P, (b) the acceleration of P, when t = 2.

$v = \frac{ds}{dt} = -3t^2 + 12t - 5$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = -6t + 12$

When t = 2: v = 7 and a = 0

Answer: (a) 7 m/s (b) 0 m/s^2

2 Recurring decimals

Write $0.3\dot{2}7\dot{1}$ as a fraction in its simplest form.

Let x = 0.327127...

10x = 3.271271...

10,000x = 3271.271271...

Subtract: 9990x = 3268

$x = \frac{3268}{9990} = \frac{1634}{4995}$

Answer: $\frac{1634}{4995}$

3 Using a line of best fit

The scatter graph shows distance from town (x) and price (y).

The equation of the line of best fit is $y = -2.5x + 34$.

(a) Estimate y when x = 7.

(b) Explain why an estimate of y when x = 25 may be unreliable.

(a) $y = -2.5 \times 7 + 34 = 16.5$

(b) x = 25 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)

Answer: (a) 16.5 (b) x = 25 is outside the range of the data

4 Standard form: multiplying and dividing

Work out the square root of 4×10^{-2}.

Give your answer in standard form.

Write 4×10^{-2} as 4×10^{-2} (even power of 10)

Square root: $\sqrt{4} = 2$ and $\sqrt{(10^{-2})} = 10^{-1}$

$= 2 \times 10^{-1}$

Answer: 2×10^{-1}

5 Two numbers: maximise or minimise

The positive numbers x and y satisfy x + y = 3.

Find the greatest possible value of x^2y, showing that it is the greatest.

$y = 3 - x$, so $x^2y = -x^3 + 3x^2$

$\frac{d}{dx}(x^2y) = -3x^2 + 6x = 0$, so $x(6 - 3x) = 0$ and $x = 2$ $(x > 0)$

Second derivative $= -6x + 6 = -6 < 0$, so a maximum

$y = 1$, so the greatest value is $2^2 \times 1 = 4$

Answer: 4

6 Hollow cylinder and density

A metal pipe is a hollow cylinder with outer radius 10 cm and inner radius 9.5 cm. (Calculator)
The density of the metal is 2.7 g/cm^3.
The mass of the pipe is 3.31 kg.
Work out the length of the pipe. Give your answer in centimetres correct to 3 significant figures.

Mass = 3310 g, so volume = mass ÷ density = 3310 ÷ 2.7 = 1225.9 cm^3
Cross-sectional area = $\pi \times 10^2 - \pi \times 9.5^2 = 30.631$ cm^2
Length = volume ÷ area = 1225.9 ÷ 30.631 = 40.0 cm (3 s.f.)
Answer: 40.0 cm

7 Direct and inverse proportion

P is inversely proportional to t.
When t = 2, P = 3.
(a) Find a formula for P in terms of t.
(b) Find the value of P when t = 3.

(a) $P = \frac{k}{t}$

Substitute t = 2 and P = 3: $3 = \frac{k}{2}$

k = 6

$P = \frac{6}{t}$

(b) $P = \frac{6}{3} = 2$

Answer: (a) $P = \frac{6}{t}$ (b) 2

8 Cylinder of fixed surface area

A closed cylinder (a tin) has radius r cm, height h cm and total surface area 294π cm^2.
(a) Show that its volume, V cm^3, is $V = 147\pi r - \pi r^3$.
(b) Find the maximum value of V in terms of π, and show that h = 2r when V is a maximum.

$2\pi r^2 + 2\pi rh = 294\pi$, so $h = \frac{147 - r^2}{r}$

$V = \pi r^2 h = \pi r(147 - r^2) = 147\pi r - \pi r^3$

$\frac{dV}{dr} = 147\pi - 3\pi r^2 = 0$, so $r^2 = 49$ and r = 7

$\frac{d^2V}{dr^2} = -6\pi r < 0$, so a maximum. $h = \frac{147 - 49}{7} = 14 = 2r$

$V = \pi \times 7^2 \times 14 = 686\pi$
Answer: 686π cm^3

9 Variation: missing values

p is directly proportional to q.
When q = 5, p = 30.
When q = 6, p = a.
When q = b, p = 48.
Work out the values of a and b.

p = kq, k = 30 ÷ 5 = 6
a = 6 × 6 = 36
48 = 6b so b = 8, b = 8
Answer: a = 36, b = 8

10 Bounds and accuracy

a = 6.21 and b = 61.5, both correct to 3 significant figures.
Work out a × b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

$6.205 \leq a < 6.215$; $61.45 \leq b < 61.55$
Lower bound = 6.205 × 61.45 = 381.29725
Upper bound = 6.215 × 61.55 = 382.53325
Both bounds round to 380 to 2 significant figures, but not to 3 significant figures
Answer: 380 (2 s.f.)