Day 63 **IGCSE 10-a-day: Model Solutions**

1 Solving trigonometric equations

Solve $3\cos x - 2 = 0$ for $0° \leq x \leq 360°$. (Calculator)
Give your answers correct to 1 decimal place.

$\cos x = \frac{2}{3}$

Related acute angle: $\cos^{-1}(\frac{2}{3}) = 48.2°$

cosine is positive in the 1st and 4th quadrants, so $x = 48.2°$ and $x = 360° - 48.2°$
$x = 48.2°$ and $x = 311.8°$
Answer: $x = 48.2°$ or $x = 311.8°$

2 Error intervals

The height of a tower is 56 m, truncated to a whole number.
Write down the error interval for the height of a tower, using h.

Truncating cuts off the later digits without rounding.
The smallest possible value is 56 and the value must be less than 57
$56 \leq h < 57$
Answer: $56 \leq h < 57$

3 Turning point of a quadratic by differentiation

$f(x) = 3x^2 + 24x - 1$
Use differentiation to find the coordinates of the turning point of the graph of $y = f(x)$. State whether it is a maximum or a minimum.

$f'(x) = 6x + 24$
$f'(x) = 0$: $6x + 24 = 0$, so $x = -4$
$f(-4) = -49$
$f''(x) = 6$, which is positive, so it is a minimum
Answer: $(-4, -49)$, minimum

4 Inverse variation

W is inversely proportional to h.
When h = 10, W = 1.2.
(a) Find a formula for W in terms of h.
(b) Find W when h = 15.

$W = \frac{k}{h}$

$1.2 = \frac{k}{10}$ so $k = 1.2 \times 10 = 12$

$W = \frac{12}{h}$

$W = \frac{12}{15} = 0.8$

Answer: (a) $W = \frac{12}{h}$ (b) $W = 0.8$

5 Local maximum and minimum values

$f(x) = x^3 - 27x + 2$
Find the y coordinate of the local maximum point and the y coordinate of the local minimum point.

$f'(x) = 3x^2 - 27 = 3(x - 3)(x + 3)$
$f'(x) = 0$ gives $x = -3$ or $x = 3$
$f''(x) = 6x$: $f''(-3) = -18 < 0$ (maximum) and $f''(3) = 18 > 0$ (minimum)
$f(-3) = 56$ and $f(3) = -52$
Answer: Local maximum $y = 56$, local minimum $y = -52$

6 Transformed curve from a diagram

The diagram shows the graph of y = f(x) with a minimum point at (1, 2).
The dashed curve C is the image of y = f(x) after a translation.
The minimum point of C is (5, 0).
Write down the equation of C in terms of f.

The minimum point has moved from (1, 2) to (5, 0)

Translation by $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$

x moves right by 4: f(x − 4). y moves down by 2: − 2

Answer: y = f(x − 4) − 2

7 Algebraic fractions: adding and subtracting

Show that $\frac{x+3}{2x-3} + \frac{6}{x-1}$ can be written as $\frac{x^2+14x-21}{(2x-3)(x-1)}$

The common denominator is (2x − 3)(x − 1)

$$\frac{x+3}{2x-3} + \frac{6}{x-1} = \frac{(x+3)(x-1)+6(2x-3)}{(2x-3)(x-1)}$$

Expand the numerator: $x^2 + 14x - 21$

$$\frac{x^2+14x-21}{(2x-3)(x-1)}$$

Answer: $\mathbf{\frac{x^2+14x-21}{(2x-3)(x-1)}}$

8 Maximum profit

A company makes and sells x thousand items. All amounts are in thousands of pounds.
The revenue is R = x(36 − 3x) and the cost is C = 9 + 6x.
(a) Show that the profit P = R − C is P = $-3x^2 + 30x - 9$.
(b) Find the number of items that must be sold for the profit to be a maximum, and this maximum profit.

$P = 36x - 3x^2 - (9 + 6x) = -3x^2 + 30x - 9$

$\frac{dP}{dx} = -6x + 30 = 0$ gives x = 5

$\frac{d^2P}{dx^2} = -6 < 0$, so a maximum

P = 66

Answer: 5000 items, maximum profit £66000

9 Ratio and mixtures

Alloy A holds copper and tin in the ratio 4 : 5.
Alloy B holds copper and tin in the ratio 4 : 2.
18 kg from alloy A and 24 kg from alloy B are mixed.
Work out the ratio of copper to tin in the mixture.

From A: copper = $18 \times \frac{4}{9}$ = 8 kg, tin = 10 kg

From B: copper = $24 \times \frac{2}{3}$ = 16 kg, tin = 8 kg

copper: 8 + 16 = 24 and tin: 10 + 8 = 18

24 : 18 = 4 : 3

Answer: 4 : 3

10 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle APB = 78°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angles in quadrilateral OAPB: 360 − 90 − 90 − 78 = 102°
(b) The angle at the circumference is half the angle at the centre: 102 ÷ 2 = 51°
Answer: (a) 102° (b) 51°