Day 62 **IGCSE 10-a-day: Model Solutions**

1 Coordinates and gradient at the y-intercept

$y = -2x^4 - 3x^3 + 6x^2 - 5x - 9$
The curve crosses the y-axis at the point A.
Work out the coordinates of A and the gradient of the curve at A.

When $x = 0$, $y = -9$, so A is $(0, -9)$

$\frac{dy}{dx} = -8x^3 - 9x^2 + 12x - 5$

When $x = 0$, $\frac{dy}{dx} = -5$

Answer: A = (0, −9), gradient −5

2 Mean of a changing set

The mean of 5 numbers is 57.
One number is removed and the mean of the remaining 4 numbers is 56.5.
Work out the number that was removed.

Total of the 5 numbers = $5 \times 57 = 285$
Total of the 4 remaining = $4 \times 56.5 = 226$
Number removed = $285 - 226 = 59$
Answer: 59

3 Arc length and sector area

A sector of a circle has angle 135° and area 6π cm^2.
(a) Work out the radius of the circle.
(b) Work out the length of the arc of the sector. Give your answer to (b) in terms of π.

(a) Area = $\frac{135}{360} \times \pi \times r^2$, so $\frac{135}{360} \times r^2 = 6$

$r^2 = 6 \times \frac{360}{135} = 16$

$r = 4$ cm

(b) Arc length = $\frac{135}{360} \times 2 \times \pi \times 4 = 3\pi$ cm

Answer: (a) 4 cm (b) 3π cm

4 Inverse of a linear function

$f(x) = -5x - 6$.
(a) Find $f^{-1}(x)$.
(b) Find $f^{-1}(-5)$.

(a) Let $y = -5x - 6$, then rearrange to make x the subject

$x = \frac{-6 - y}{5}$, so $f^{-1}(x) = \frac{-6 - x}{5}$

(b) $f^{-1}(-5) = -\frac{1}{5}$

Answer: (a) $f^{-1}(x) = \frac{-6 - x}{5}$ (b) $-\frac{1}{5}$

5 Product of brackets: tangent gradient

$y = x(x - 1)(2x + 1)$
Work out the gradient of the curve at the point where $x = 2$.

Expand: $y = 2x^3 - x^2 - x$

$\frac{dy}{dx} = 6x^2 - 2x - 1$

$x = 2$: $\frac{dy}{dx} = 19$

Answer: 19

6 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve with a turning point at (−2, 2).
The curve crosses the y-axis at (0, −2).
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The turning point is (−2, 2), so $y = a(x + 2)^2 + 2$
When x = 0, y = −2: $a \times 4 + 2 = -2$, so $a = -1$
$y = -(x + 2)^2 + 2$
Expand: $y = -x^2 - 4x - 2$
Answer: $y = -x^2 - 4x - 2$

7 Compound interest: how many years

£4,000 is invested at 10% per year compound interest.
Work out the least whole number of years after which the investment is worth more than £7,000. (Calculator)

Value after n years = 4000×1.1^n
After 5 years: £6,442.04 (less than £7,000)
After 6 years: £7,086.24 (more than £7,000)
Answer: 6 years

8 Equal roots and real roots

Show that the equation $x^2 + 2kx - 2 = 0$ has two distinct real solutions for all values of k.

$b^2 - 4ac = (2k)^2 - 4 \times 1 \times (-2)$
$= 4k^2 + 8$
$4k^2$ is never negative, so $4k^2 + 8$ is always at least 8, which is greater than 0
So $b^2 - 4ac > 0$ for all k, and there are always two distinct real solutions
Answer: Shown

9 Fractional indices

Given that x > 0, solve $x^{\frac{5}{3}} = 243$

Raise both sides to the power $\frac{3}{5}$

$x = (243)^{\frac{3}{5}}$
x = 27
Answer: x = 27

10 Distance travelled

The diagram shows the graph of $s = -t^3 + 12t^2 - 45t$, the displacement in metres of a particle P from O after t seconds.
P changes direction at A and B. Work out the total distance travelled in the first 7 seconds.

$v = \frac{ds}{dt} = -3t^2 + 24t - 45 = 0$ at t = 3 and t = 5 (A and B)
s(0) = 0, s(3) = −54, s(5) = −50, s(7) = −70
Distances: $|-54 - 0| + |-50 - (-54)| + |-70 - (-50)|$
$= 54 + 4 + 20 = 78$
Answer: 78 m