Day 61 **IGCSE 10-a-day: Model Solutions**

1 Gradient at a point

Work out the gradient of the curve $y = x^3 + 3x^2 + x$ at the point where $x = -3$.

$\frac{dy}{dx} = 3x^2 + 6x + 1$

When $x = -3$: gradient = 10

Answer: 10

2 Magnitude of a vector

$\mathbf{v} = \begin{pmatrix} k \\ 4 \end{pmatrix}$ and $|\mathbf{v}| = 5$.

Find the two possible values of k.

$k^2 + 16 = 25$, so $k^2 = 9$

$k = 3$ or $k = -3$

Answer: k = 3 or k = −3

3 Rates of work

Anna can paint a fence in 20 hours.
Ben can paint the same fence in 60 hours.
They work together at these rates.
How long do they take to paint the fence?

In 1 hour: Anna does $\frac{1}{20}$ and Ben does $\frac{1}{60}$

Together: $\frac{1}{20} + \frac{1}{60} = \frac{80}{1200}$

Time = $\frac{1200}{80}$ = 15 hours

Answer: 15 hours

4 Stationary points of a cubic

A curve has equation $y = -2x^3 + 6x^2 + 4$.
Find the coordinates of the stationary points of the curve.

$\frac{dy}{dx} = -6x^2 + 12x$

Stationary points: $\frac{dy}{dx} = 0$, so $-6x(x - 2) = 0$

$x = 0$ or $x = 2$

$y = 4$ when $x = 0$ and $y = 12$ when $x = 2$

Answer: (0, 4) and (2, 12)

5 Exact lengths and areas

A right-angled triangle has base $(3 + \sqrt{3})$ cm and height $(3 - \sqrt{3})$ cm.
Show that the area of the triangle is a rational number.
Work out the area.

Area = $\frac{1}{2} \times (3 + \sqrt{3})(3 - \sqrt{3})$

$(3 + \sqrt{3})(3 - \sqrt{3}) = 9 - 3 = 6$

Area = $\frac{1}{2} \times 6 = 3$

Answer: 3 cm^2

6 Iteration

Use the iteration formula $x_{n+1} = 1 + \frac{5}{x_n}$ with $x_0 = 2$ (Calculator)

to work out x_1, x_2 and x_3.

Give each answer correct to 3 decimal places.

$x_1 = 1 + \frac{5}{2} = 3.50000...$

$x_2 = 1 + \frac{5}{x_1} = 2.42857...$

$x_3 = 1 + \frac{5}{x_2} = 3.05882...$

Rounded: $x_1 = 3.500$, $x_2 = 2.429$, $x_3 = 3.059$

Answer: $x_1 = 3.500$, $x_2 = 2.429$, $x_3 = 3.059$

7 Probabilities with an unknown

A biased spinner lands on one of four outcomes with these probabilities.

red: 2x; blue: 3x; green: 3x + 0.1; yellow: 0.1

(a) Work out the value of x.

(b) The biased spinner is used 200 times. Work out an estimate for the number of times it lands on red.

(a) Probabilities add up to 1: 2x + 3x + 3x + 0.1 + 0.1 = 1

8x + 0.2 = 1, so 8x = 0.8

x = 0.1

(b) P(red) = 2 × 0.1 = 0.2, so 200 × 0.2 = 40

Answer: (a) x = 0.1 (b) 40

8 Intersecting lines in a triangle

$\mathbf{OA} = \mathbf{a}$, $\mathbf{OB} = \mathbf{b}$, $\mathbf{OP} = \frac{2}{3}\mathbf{a}$ and $\mathbf{OQ} = \frac{1}{4}\mathbf{b}$.

AQ and BP meet at X.

(a) Find $\mathbf{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) Find the ratio AX : XQ.

Let $\mathbf{AX} = s\mathbf{AQ}$. Then $\mathbf{OX} = \mathbf{a} + s(-\mathbf{a} + \frac{1}{4}\mathbf{b}) = (1 - s)\mathbf{a} + \frac{1}{4}s\mathbf{b}$

Let $\mathbf{BX} = t\mathbf{BP}$. Then $\mathbf{OX} = \mathbf{b} + t(\frac{2}{3}\mathbf{a} - \mathbf{b}) = \frac{2}{3}t\mathbf{a} + (1 - t)\mathbf{b}$

Compare coefficients: $1 - s = \frac{2}{3}t$ and $\frac{1}{4}s = 1 - t$

Solving gives $s = \frac{2}{5}$ and $t = \frac{9}{10}$

(a) $\mathbf{OX} = \frac{3}{5}\mathbf{a} + \frac{1}{10}\mathbf{b}$

(b) AX = $\frac{2}{5}$ × AQ, so AX : XQ = $\frac{2}{5} : \frac{3}{5} = 2 : 3$

Answer: (a) $\frac{3}{5}\mathbf{a} + \frac{1}{10}\mathbf{b}$ (b) 2 : 3

9 Area of a triangle using sine

In triangle PQR, PQ = 8 cm and angle PQR = 150°.
The area of the triangle is 26 cm^2.
Work out the length of QR.
Do not use a calculator.

Area = $\frac{1}{2} \times PQ \times QR \times \sin Q$

$26 = \frac{1}{2} \times 8 \times QR \times \sin 150°$

$\sin 150° = \frac{1}{2}$, so $26 = 2 \times QR$

$QR = 26 \div 2 = 13$ cm

Answer: 13 cm

10 Cylinder of fixed surface area

A closed cylinder (a tin) has radius r cm, height h cm and total surface area 150π cm^2.
(a) Show that its volume, V cm^3, is $V = 75\pi r - \pi r^3$.
(b) Find the maximum value of V in terms of π, and show that $h = 2r$ when V is a maximum.

$2\pi r^2 + 2\pi rh = 150\pi$, so $h = \frac{75 - r^2}{r}$

$V = \pi r^2 h = \pi r(75 - r^2) = 75\pi r - \pi r^3$

$\frac{dV}{dr} = 75\pi - 3\pi r^2 = 0$, so $r^2 = 25$ and $r = 5$

$\frac{d^2V}{dr^2} = -6\pi r < 0$, so a maximum. $h = \frac{75 - 25}{5} = 10 = 2r$

$V = \pi \times 5^2 \times 10 = 250\pi$

Answer: 250π cm^3